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AND

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AMERICAN SOCIETY OF CIVIL ENGINEERS

Founded November 5, 1852

PAPERS

DESIGN LIVE LOADS IN BUILDINGS

By John W. Dunham,[1] M. ASCE

Synopsis

Based upon surveys of the actual live loads in office buildings and warehouses, and upon a consideration of probable actual live loads in apartment buildings and places of assembly, this paper develops and proposes a system for the reduction of basic live loads in the design of building members.

1. Introduction

General.—It is common practice to reduce basic live loads for use in the design of a particular member depending in some fashion on the size of the area supported. For columns this reduction has commonly been a function of the number of floors carried. For beams, when permitted, the reduction has been a function of the tributary area. Thus the general principle is well recognized.

Quantitative information that defines the relation between the probable average live load and the size of the loaded area in the various occupancies is difficult to find. Some pertinent information for office buildings has been published in the form of live-load surveys.[2] Such data are valuable but cover only limited parts of buildings. They do suggest, however, that larger reductions of live load than those ordinarily used may be permissible.

In order to obtain factual information bearing upon this relationship, the U. S. Public Buildings Administration surveyed the live loads in the southeast quarter of the Internal Revenue Building and the main portion of the Veterans Administration Building, both in Washington, D. C., and in eight private warehouses. The surveys were made under the general direction of the writer.

This paper is based upon a report, "Live Loads in Office Buildings and Warehouses," prepared by the writer in the structural engineering division of the U. S. Public Buildings Administration, In addition, it treats the question of live-load reduction in apartment buildings and assembly occupancies and proposes a general system of live-load reduction.

Definitions.—For the purposes of this paper the following terms will be assumed to have the meanings indicated:

Note.—Written comments are invited for immediate publication; to insure publication the last discussion should be submitted by September 1, 1946.

[1] Asst. Chf. Structural Engr., Public Bldgs. Administration, Washington, D. C.

[2] *Engineering News-Record*, March 29, 1923, p. 584.

Basic Live Load.—The live load per square foot of floor area for which the floor is designed or may be posted.

Unreduced Live Load.—The product of the loaded area under consideration and the basic live load.

Actual Live Load.—The total of all the live load found in a given area.

Average Actual Live Load.—The total live load found in a given area divided by that area.

Design Live Load.—The product of the area under consideration and the basic live load for that area reduced as permitted by a design specification. This is the load that a member of a structure is designed to carry.

2. Design Live Loads in Office Buildings

Method of Conducting Surveys.—The average live load for each panel of floor including that in the corridors was obtained and recorded on a set of plans. The live load included furniture and its contents, movable property generally, and the people found in each space at the time it was surveyed. Weights were determined as follows: (*a*) Representative pieces of furniture, and their normal contents, were weighed with spring scales—the weights thus determined being used for all corresponding items found in the survey of the building; (*b*) persons found in a given panel at the time it was surveyed were counted and assumed to weigh 150 lb each; (*c*) special furniture and equipment not corresponding to the typical items were weighed with spring scales in the space in which they were found. In some cases the pieces were too large to weigh in their entirety and in these instances parts of the equipment, such as separate drawers, were weighed and the weight of the entire unit computed; and (*d*) weights of miscellaneous items such as rugs, pictures, and papers on top of filing cabinets were estimated.

Internal Revenue Building.—A survey was made of the live loads in the southeast section of the Internal Revenue Building (identified herein as I R Building), Washington, D. C. This part of the building is approximately 226 ft by 210 ft in plan and seven stories high, plus an unfinished attic story which is omitted from this analysis. There is a central court about 135 ft by 110 ft in plan extending through all stories. The average usable area of each floor in this portion of the building is approximately 26,000 sq ft. There are seventy columns in this area. The basic live load used in the design of the building is 100 lb per sq ft. No reduction was used for the floor slabs or beams. The design live load on the columns was obtained by considering the seventh floor to be the top and using the following percentages of the basic live load: 100% on the seventh floor; 90% on the sixth floor; 80% on the fifth floor; 70% on the fourth floor; 60% on the third floor; and 50% on each of the remaining floors.

Veterans Administration Building.—A survey was made of the live loads in the main part of this building (V A Building), which is L-shaped, 60 ft wide, and approximately 462 ft in developed length. There is also an attached wing 44 ft by 51 ft. It is eleven stories high above grade and there are 144 columns extending through all stories. The area of each floor in this part of the building including the wing is about 29,000 sq ft.

This building was not designed by the federal government but was purchased for governmental use. There is some doubt regarding the design live loads; however, for purposes of comparison, a basic live load of 70 lb per sq ft. has been assumed. An assumed design live load on the columns was obtained by using the following percentages of the basic live load: 100% on the eleventh floor; 90% on the tenth floor; 80% on the ninth floor; 70% on the eighth floor; 60% on the seventh floor; and 50% on the remaining floors.

Statistical Distribution of Live Load.—The total areas occupied by average, actual live loads of various ranges are given in Table 1, both for individual floors

TABLE 1.—DISTRIBUTION OF ACTUAL UNIT LIVE LOADS, BY AREAS

Floor No.	Live Load,[a] in Pounds per Square Foot: 0 to 20		20 to 40		40 to 60		60 to 80		80 to 100		100 to 110		Total
	Sq ft	%	Sq ft	%	Sq ft	%	Sq ft	%	Sq ft	%	Sq ft	%	Sq ft
	(*a*) Internal Revenue Building												
1	19,814	74	5,229	20	890	3	790	3	....	....	....	....	26,723
2	23,999	90	1,988	7	....	....	....	....	....	....	825	3	26,812
3	18,449	70	5,040	19	1,030	4	1,100	4	770	3	....	....	26,389
4	21,367	81	4,965	19	....	....	....	....	....	....	....	....	26,332
5	25,480	97	845	3	....	....	....	....	....	....	....	....	26,325
6	7,850	30	9,966	39	6,830	26	1,290	5	....	....	....	....	25,936
7	10,290	45	4,676	21	6,412	28	1,263	6	....	....	....	....	22,641
Total	127,249	(70)	32,709	(18)	15,162	(8.5)	4,443	(2.5)	770	(0.5)	825	(0.5)	181,158
	(*b*) Veterans Administration Building												
1	27,992	97.5	625	2.5	....	....	....	....	....	....			28,617
2	27,996	94.5	896	3	625	2.5	....	....	....	....			29,517
3	22,123	75	3,380	11	4,014	14	....	....	....	....			29,517
4	29,517	100	....	....	....	....	....	....	....	....			29,517
5	27,621	93.5	1,896	6.5	....	....	....	....	....	....			29,517
6	28,997	98	260	1	260	1	....	....	....	....			29,517
7	29,005	98	512	2	....	....	....	....	....	....			29,517
8	28,587	96.5	....	....	930	3.5	....	....	....	....			29,517
9	29,517	100	....	....	....	....	....	....	....	....			29,517
10	27,365	92.5	976	3.5	....	....	....	....	1,176	4			29,517
11	23,188	98	509	2	....	....	....	....	....	....			23,697
Total	301,908	(95)	9,054	(2.75)	5,829	(1.75)	....	....	1,176	(0.5)			317,967

[a] The percentages opposite each floor refer to the indicated total area of that floor. Those in parentheses, opposite to "Total" line, refer to the total area of the building.

and for each building as a whole. For instance, on the first floor of the I R Building 5,229 sq ft are occupied by average, actual live loads varying from 20 lb per sq ft to 40 lb per sq ft. For the building as a whole, 32,709 sq ft are occupied by actual live loads in this range. Of the total floor area of the building 88% supported an average, actual live load of 40 lb per sq ft, or less. Of the total floor area of the V A Building 99.5% supported an actual, average live load of 60 lb per sq ft or less, and 97.75% supported an actual, average live load of 40 lb per sq ft or less.

Variation of Live Load on Floors.—The maximum, average, actual, unit live load found in the I R Building occurred on 825 sq ft of the second floor (in the V A Building, 1,176 sq ft of the tenth floor) and amounted to 106 lb per sq ft (90 lb per sq ft in the V A Building). To show how the average, actual live

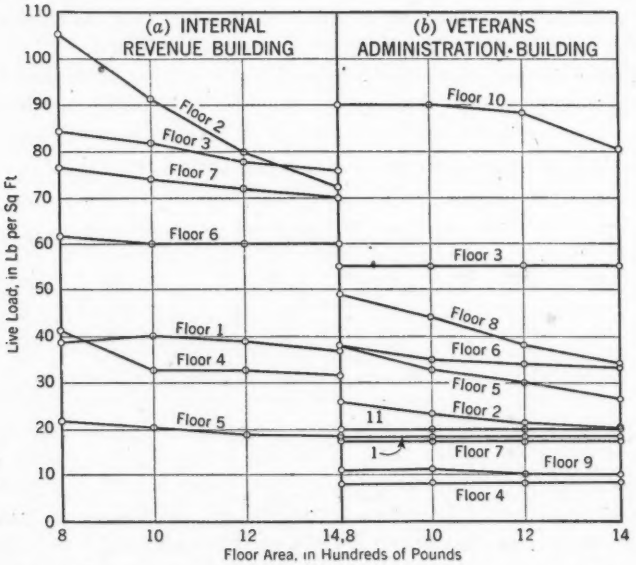

FIG. 1.—AVERAGE ACTUAL UNIT LIVE LOAD ON THE MOST HEAVILY LOADED FLOOR AREAS

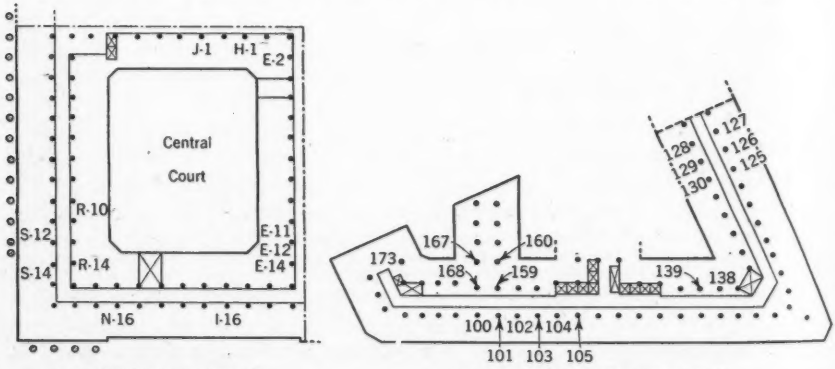

FIG. 2.—LOCATION OF BUILDING COLUMNS

load varied with respect to area in the most heavily loaded regions, the continuous areas of 800, 1,000, 1,200, and 1,400 sq ft carrying the greatest average, actual live load were determined for each floor. The results have been plotted for each building in Fig. 1.

Variation of Live Load on Columns.—To show how the actual column live loads in the I R Building varied in relation to accumulated tributary area, the twelve columns (nineteen columns in the V A Building) which, from inspection, seemed to be the most heavily loaded were selected (see Fig. 2). These columns

TABLE 2.—VARIATION OF TOTAL, ACTUAL, LIVE LOAD ON COLUMNS
(See Fig. 2 for Location of Columns)

Floor No.	Tributary Area (Sq Ft)		Unreduced Live Load, in Kips		Actual Live Load						Design Load	
					Percentage of:				Percentage of:			
	One floor	Cumulative	One floor	Cumulative	Col. 4	Col. 5	One floor	Cumulative	Col. 12	Col. 13	One floor	Cumulative
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)
					(*a*) Internal Revenue Building, Column E-2							
7	292	292	29.2	29.2	56	56	16.4	16.4	56	56	29.2	29.2
6	292	584	29.2	58.4	30	42	8.1	24.5	31	45	26.3	55.5
5	292	876	29.2	87.6	14	32	4.0	28.5	17	36	23.3	78.8
4	292	1,168	29.2	116.8	29	32	8.5	37.0	42	37	20.4	99.2
3	292	1,460	29.2	146.0	75	40	21.9	58.9	125	50	17.5	116.7
2	292	1,752	29.2	175.2	7	35	2.1	61.0	14	46	14.6	131.3
1	292	2,044	29.2	204.4	23	23	6.6	67.6	45	46	14.6	145.9
					(*b*) Internal Revenue Building, Column H-1							
7	236	236	23.6	23.6	59	59	13.8	13.8	58	59	23.6	23.6
6	236	472	23.6	47.2	23	41	5.7	19.5	27	43	21.2	44.8
5	236	708	23.6	70.8	11	31	2.6	22.1	14	35	19.0	63.8
4	236	944	23.6	94.4	36	32	8.5	30.6	51	38	16.5	80.3
3	236	1,180	23.6	118.0	63	39	14.9	45.5	105	48	14.2	94.5
2	236	1,416	23.6	141.6	5	33	1.2	46.7	10	44	11.8	106.3
1	236	1,652	23.6	165.2	30	33	7.2	53.9	61	45	11.8	118.1
					(*c*) Internal Revenue Building, Column S-14							
7	214	214	21.4	21.4	47	47	10.0	10.0	47	47	21.4	21.4
6	274	488	27.4	48.8	66	58	18.1	28.1	73	61	24.7	46.1
5	274	762	27.4	76.2	13	42	3.7	31.8	17	47	22.0	68.1
4	274	1,036	27.4	103.6	19	36	5.3	37.1	28	42	19.2	87.3
3	274	1,310	27.4	131.0	6	30	1.6	38.7	10	37	16.5	103.8
2	274	1,584	27.4	158.4	8	26	2.2	40.9	16	35	13.7	117.5
1	274	1,858	27.4	185.8	19	25	5.3	46.2	39	35	13.7	131.2
					(*d*) Veterans Administration Building, Column 138							
11	274	274	19.18	19.18	17	17	3.20	3.20	17	17	19.18	19.18
10	274	548	19.18	38.36	129	73	24.70	27.90	143	77	17.30	36.48
9	274	822	19.18	57.54	9	52	1.71	29.61	11	57	15.40	51.88
8	274	1,096	19.18	76.72	19	43	3.56	33.17	26	51	13.50	65.38
7	274	1,370	19.18	95.90	21	39	4.07	37.24	35	48	11.50	76.88
6	274	1,644	19.18	115.70	20	35	3.92	41.16	41	48	9.60	86.48
5	274	1,918	19.18	134.88	34	35	6.46	47.62	67	49	9.60	96.08
4	274	2,192	19.18	154.06	14	33	2.62	50.24	26	48	9.60	105.68
3	274	2,466	19.18	173.24	14	30	2.73	52.97	28	46	9.60	115.28
2	274	2,740	19.18	192.42	22	30	4.31	57.28	45	46	9.60	124.88
1	274	3,014	19.18	211.60	2	27	0.41	57.69	4	43	9.60	134.48

constitute more than one sixth of the columns in the part of the I R Building involved. The actual live load supported by each of these columns at each floor and its accumulation in each story were computed. The results for the critical columns in both buildings are given in Cols. 8 and 9 of Table 2. The

three columns of the I R Building that supported the highest, average, actual live loads (each for part of its length) are reported in Tables 2(*a*), 2(*b*), and 2(*c*). Table 2(*d*) includes the corresponding data for that column of the V A Building of the nineteen investigated, which supported the greatest, average, actual live load. Using a basic live load of 100 lb per sq ft (70 ft per sq ft in the V A Building), the unreduced live loads for each of these columns were computed and are recorded in Cols. 4 and 5. The actual live loads expressed as percentages of the corresponding unreduced live loads are given in Cols. 6 and 7.

In order to show, graphically, the relation between the average, actual live load supported by each story of the column and the accumulated area supported by that story the corresponding values for several representative columns of both buildings were plotted in Fig. 3. It will be noted in the I R Building that

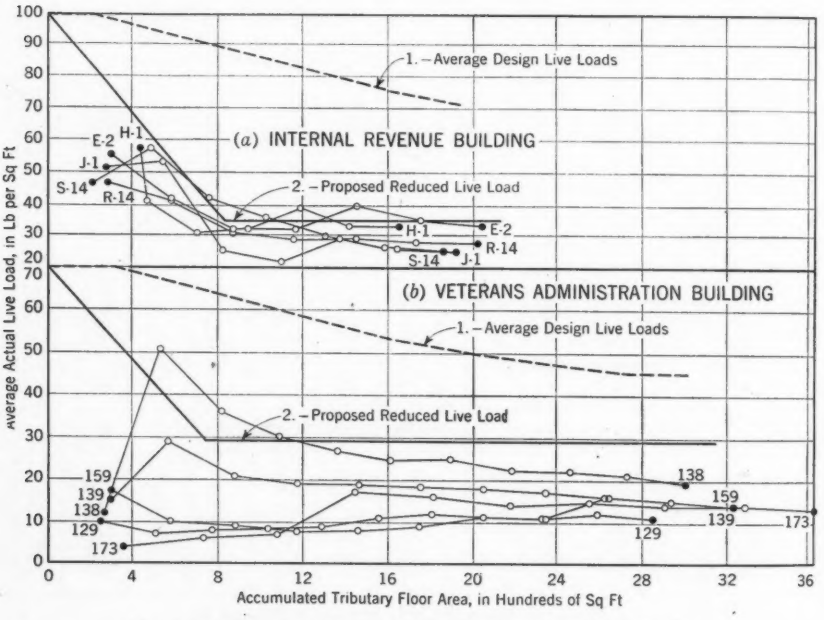

FIG. 3.—VARIATION OF LIVE LOAD ON THE COLUMNS OF BUILDINGS (FOR IDENTITY OF COLUMNS SEE FIG. 2)

no one column is critical but that different columns are critical for various areas. It appears that the intensity of live load diminishes faster on columns as the area increases than it does on floors.

Significance of Survey Data.—A consideration of the loadings found in these surveys and their comparison with those previously reported[2] indicates that the I R Building is representative of a rather heavy occupancy and that the V A Building is representative of an average occupancy, although the maximum load found in the latter is higher than appears usual.

The surveys show clearly that, as the tributary area of a member increases, the chances of its being fully loaded decrease. The decrease in actual live load

was more marked in the case of columns than in that of floors. It appears to be a fact that the probability of several contiguous floor panels being fully loaded is much greater than that several successive stories of a column will be fully loaded.

One of the common ways of reducing the live loads on columns heretofore has been to use the following percentages of the basic live load: 100% on the top floor; 90% on the first floor below the top; 80% on the second floor below the top; 70% on the third floor below the top; 60% on the fourth floor below the top; and 50% on the other floors.

In order to compare the results obtained by this method with the loads that actually occurred in the buildings surveyed, the heavy dotted lines (curves 1, Fig. 3) were plotted in Figs. 3(*a*) and 3(*b*) to show the average design loads with respect to area, using the foregoing method of reduction, as applied to columns carrying 275 sq ft of floor. For this purpose basic live loads of 100 lb per sq ft and 70 lb per sq ft for the I R Building and the V A Building, respectively, have been used. It is clear from a comparison of the heavy dotted lines and the loads actually found that the correspondence is not good and that the shape of the line is not that of the actual loading.

It is noteworthy that the maximum live loads found in each case occurred in only one part of the building and that this part was less than 1% of the floor area of the building.

There is an obvious advantage, as regards flexibility of use, to design the building so that these maximum loads can be permitted anywhere. There is just as obviously a lack of economy in providing strength throughout the structure that will not be used in 99% of the building.

Therefore, it seems logical, if possible, to design an office building in accordance with a calculated risk of occasional overstress in the members, limited so as to insure against the possibility of serious overstress. If this can be done the building can be posted for its basic live load with the assurance that, even if any member is fully loaded, disaster is impossible.

Basic Live Loads and Design Live Loads.—Such a system of basic and design live loads is possible. It should conform to the following essentials: (*a*) A basic live load equal to the maximum to be allowed on any panel; (*b*) a rapid reduction to a minimum load based upon the relation of live and dead loads in such a manner that overstress on any member, due to full basic load throughout the structure, cannot exceed a predetermined value; and (*c*) the system should be such that only a very small portion of the structure will be overstressed by the probable actual live loads.

It is proposed that: (1) The basic live load shall be the maximum average live load that it is intended to allow on any panel; (2) the basic live load may be reduced for the design of any member at the rate of $\Delta w_L = 0.08\%$ per sq ft of tributary area; and (3) the maximum reduction shall not exceed Δw_L as determined by the following formula:

$$\Delta w_L = \frac{w_D + w_L}{4.33\ w_L} \qquad (1)$$

in which: w_D is the dead load per square foot of tributary area; and w_L is the basic live load. To express the reduction as a percentage, Eq. 1 is multiplied

by 100. The maximum reduction Δw_L is such that, if a member has a tributary area large enough to allow the maximum reduction, and if the member is loaded with the full basic live load, the overstress will not exceed 30%. In other words,

$$w_L + w_D = 1.30\,[w_D + (1 - \Delta w_L)\,w_L] = 1.3\,w_D + 1.3\,w_L - 1.3\,w_L\,\Delta w_L \quad (2)$$

Simplifying and solving for Δw_L in Eq. 2 yields Eq. 1 as a proportion; multiplying by 100 expresses the reduction as a percentage.

The proposed system, then, conforms to two of the aforementioned essentials. The question as to whether it conforms to the third, "that only a very small portion of the structure will be overstressed by the probable actual live loads," can be best determined by comparison with the loads found in the surveys. The proposed rule is illustrated graphically by the heavy, solid line curve 2, in Fig. 3(*a*).

As an example, the rule can be applied to the design of the I R Building. The typical floor beam in this building has a tributary area of 335 sq ft. The live load reduction will be ($335 \times 0.08\% = 26.80\%$), say, 27%. The design live load then is 73 lb per sq ft (plus 180 lb per sq ft dead load). Reference to Table 1(*a*) shows that not more than 3.5% of all the beams in the building would be overstressed when designed for this load. The most severe load reported was 106 lb per sq ft on an area of 825 sq ft. The most severely loaded beam, therefore, would be supporting ($106 + 180 =$) 286 lb per sq ft of tributary area. Since it has been assumed to be designed for ($73 + 180 =$) 253 lb per sq ft, the overstress will be about 13%.

So far as the beams in the I R Building are concerned, from 1% to 3½% of the beams would be overstressed in some degree. Only ½% would have the maximum overstress of 13%.

An inspection of Fig. 3(*a*) indicates that the columns would be overstressed in four locations. Since there are seventy columns and since there are seven stories of each column under consideration, this represents less than 1% of the column shafts in the building, considering each story length separately. The greatest overstress will occur in Col. E-2, Fig. 2(*a*). Considering the actual live loads and dead loads on the floors, this story of the column will be required to carry ($40 + 180 =$) 220 lb per sq ft although designed for ($35 + 180 =$) 215 lb per sq ft. The overstress obviously cannot be greater than 2.4%.

The proposed system is also illustrated graphically by the heavy, solid line, curve 2, in Fig. 3(*b*). The typical floor beam in the V A Building has a tributary area of 300 sq ft. The live-load reduction will be ($300 \times 0.08\% =$) 24%. The design live load then is ($0.76 \times 70 =$) 53.2 lb per sq ft. The maximum live load found in the survey was 90 lb per sq ft (dead load 110 lb per sq ft). The most severely loaded beam would be supporting ($90 + 110 =$) 200 lb per sq ft. Since it has been assumed to be designed for ($53.2 + 110 =$) 163.2 lb per sq ft, the overstress would be about 22½%.

The live load of 90 lb per sq ft occurred on 1,176 sq ft. Since this is less than ½% of the area of the building as a whole it is apparent that less than ½% of the floor beams would be overstressed this amount. Table 1(*b*) indicates that the remainder of the beams would have no overstress.

Curve 2, Fig. 3(*b*), indicates that the maximum overstress on any story of any column would amount to about 6% in the ninth and tenth stories of column 138, Fig. 2(*b*). Since eleven stories of 144 columns are under consideration this means that only two of the story lengths out of 1,584 would be over stressed. It appears that the proposed system conforms to the third essential required of it for office occupancies.

The proposed system of basic live loads and design live loads will permit office buildings to be posted and policed for their basic live loads without: (*a*) The incorporation of so much unused strength as heretofore; (*b*) significant overstress in members from probable loadings; and (*c*) danger of structural failure in the remote contingency of full basic live loading on the entire tributary area of any member.

Proposed System Versus Lower Basic Live Loads.—It seems apparent that the questions of proper basic live loads and of proper reductions are inseparable. Another approach to the problem of obtaining as much economy as is justified is to lower the basic live loads and to allow smaller reductions than those proposed herein. For instance the report of the Building Code Committee of the U. S. Department of Commerce[3] recommended a minimum live load on floors, for office purposes, of 50 lb per sq ft. It also recommended that a percentage reduction in total column live loads be permitted as follows:

Floors supported	%
One	0
Two	10
Three	20
Four	30
Five	40
Six	45
Seven or more	50

It is pertinent, therefore, to compare these two general recommendations to determine their relative merits. The loads found in the survey of the V A Building, which averaged 11.6 lb per sq ft for the entire structure, are of the same order as those found in the surveys referred to in Section 1 under the heading "general." It will be enlightening therefore to compare the writer's proposal as it applies to that building, with the system of loads recommended by the Building Code Committee.

In Fig. 4(*a*), the heavy line represents the design loads according to the proposed system, assuming a basic live load of 70 lb per sq ft and a dead load of 110 lb per sq ft. The dotted lines represent the design loads according to the recommendations of the Building Code Committee, assuming a basic live load of 50 lb per sq ft and that the columns each carry an area of 275 sq ft per floor.

Here are two buildings, one designed for a 70-lb live load under the proposed system, and the other designed for a 50-lb live load under recommendation 2.

The slabs of the first will be designed for a higher live load. The beams of both will be designed for practically the same live loads. The upper story and

[3] "Minimum Live Loads Allowable for Use in Design of Buildings. Report of Building Code Committee," U. S. Dept. of Commerce, November 1, 1924.

the fourth, third, second, and first story columns in the first building will be designed for a pound or two more of live load per square foot than those in the second building. The ninth, eighth, seventh, and sixth story columns will be

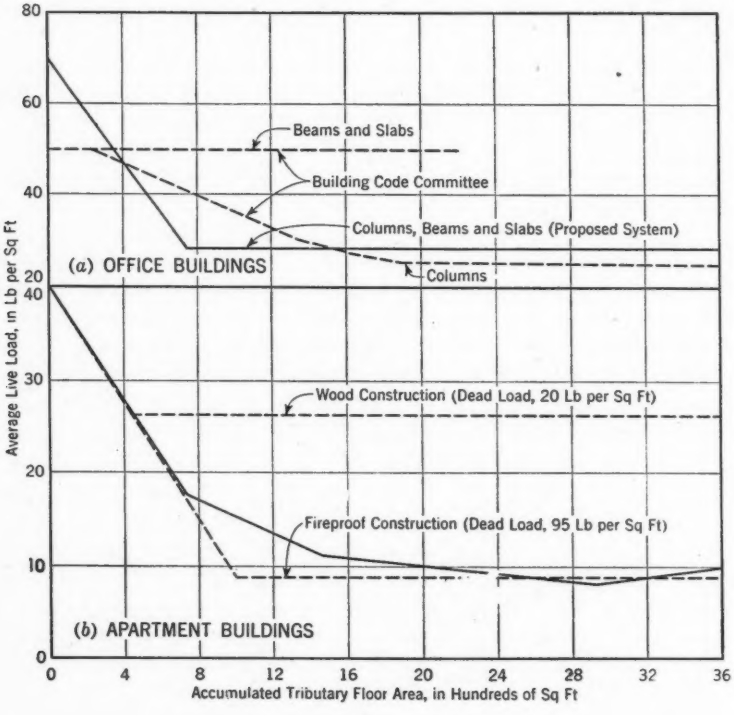

FIG. 4.—COMPARISON OF LIVE-LOAD REDUCTIONS

designed for a somewhat lower live load in the first building than in the second. The slabs in the first building will cost approximately 1½¢ more per sq ft than those of the second. The beams and columns will cost about the same in both buildings.

However, although the difference in cost is slight, the difference in usability is significant. The first building may be posted for 70 lb per sq ft of live load. The second should be posted for 50 lb per sq ft of live load if both are to fulfil the requirement that the members should not be substantially overstressed by the probable maximum live loads. The first building will be much more flexible in its use without sacrifice of safety.

It appears from the foregoing comparison that a system having a high basic live load but permitting quick reductions relative to tributary area is preferable to one using lower live loads and permitting only nominal reductions.

3. Design Live Loads in Apartment Buildings

Probable Live Loads.—The live load in apartments is made up of movable property such as furniture and furnishings, and persons. The live load created by furniture and furnishings varies from 0 to 10 lb per sq ft, with an average of about 4 lb per sq ft.[3,4] The live load attributable to persons will vary from 0 to 30 or 40 lb per sq ft.[3] These loads will be very small except in the case of special gatherings such as parties, club meetings, and funerals. A living room containing one person for every 6 sq ft is crowded, and, although denser crowds are possible, they are very unlikely because of considerations of comfort and ventilation. A crowd with one person to every 6 sq ft will average about 20 to 25 lb per sq ft. Such a crowd is very unlikely to fill more than the living room, or living room and dining room together, at any one time. It is also probable that such a crowd will occur simultaneously in comparatively few of the apartments in a building.

It will be instructive to consider an average apartment building and to discuss how the average live load may be expected to vary in relation to the area of the building under consideration.

The Bureau of Standards report cited previously[4] contains a survey made of thirteen representative apartments ranging in area from 431 sq ft to 796 sq ft. The average area was 628 sq ft. The areas of the dining room and living room in these apartments averaged about 40% of the areas of the entire apartment in each case.

To consider an example, assume that: (1) Each apartment in an apartment building has an area of 650 sq ft; (2) for each apartment there are 75 sq ft of corridor; (3) the area in each apartment subject to crowding at any one time is 260 sq ft; (4) one apartment in five will contain a crowd at the same time; (5) the crowded spaces will be loaded with 40 lb per sq ft; and (6) the remaining spaces will be loaded with 5 lb per sq ft.

TABLE 3.—Examples of Probable Live Load

Area	w_L	One Apartment		Two Apartments		Five Apartments	
		Area (sq ft)	Load (lb)	Area (sq ft)	Load (lb)	Area (sq ft)	Load (lb)
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Area subject to crowding..	40	260	10,400	260	10,400	520	20,800
Remaining areas.....	5	650+75−260	2,325	1,300+150−260	5,950	3,250+375−520	15,525
Total.....	..	725	12,725	1,450	16,350	3,625	36,325

In a single apartment and its proportionate amount of corridor, the total load will be 12,725 lb (see Table 3, Col. 4). Since the total area under consideration is 725 sq ft, the average live load is $\frac{12,725}{725} = 17.6$ lb per sq ft.

[4] "Fire Resistance Classifications of Building Construction, BMS92," Bureau of Standards Rept, 1942.

In two apartments and their 150 sq ft of corridor, the total load will be 16,350 lb (Table 3, Col. 6). The average live load will be 11.3 lb per sq ft. Similarly for four apartments and the adjoining corridor, the total load will be 23,600 lb; the area, 2,900 sq ft; and the average live load, 8.1 lb per sq ft. In five apartments there will be 520 sq ft heavily loaded so that the total load will be (Table 3, Cols. 7 and 8) 36,325 lb. The area is 3,625 sq ft and the average live load is 10.0 lb per sq ft.

Basic Live Loads and Design Live Loads.—The foregoing average live loads are plotted against the corresponding areas in Fig. 4(*b*). The line connecting the points rises slightly (as would be expected at the area representing five apartments) and if continued would fall again until an area representing ten apartments were reached.

In order to test the validity of the proposal for reducing live loads in office buildings to this type of loading, the dotted lines were drawn to show the design live loads obtained by that proposal for wood construction and fireproof construction. The relationship of the average live load to the design live load obtained by applying the proposed reduction formula to a basic live load of 40 lb per sq ft is good.

There would be no overstress in the case of the wood construction, whereas in the case of the fireproof construction the maximum probable overstress indicated is 6%. Since the maximum possible overstress with full basic live load is limited to 30% by the method it is believed that the proposed method of live-load reduction is applicable also to this type of occupancy.

4. Design Live Loads in Assembly Occupancies

Spaces in buildings to be used for assembly purposes are definitely intended to be occupied fully at times. Moreover, there is a distinct probability of impact because occasionally the people present will move simultaneously; for example, they will rise at the playing of the national anthem. It is logical, therefore, that the basic live loads for such spaces should not be reduced for the design of supporting members.

5. Design Live Loads in Warehouses

General.—Most codes do not permit the use of reduced live loads for either the columns or the beams in warehouses. So far as floor members are concerned this practice seems sound. The tributary areas are not likely to be large and it is well-known that several adjacent floor panels are frequently fully loaded.

Accordingly, the investigation of variation in live-load intensities in warehouses was limited to column loads. The investigation was conducted to obtain data bearing on the relation of average, actual live loads on columns to the accumulated tributary areas carried by the columns. Surveys were made by the Public Buildings Administration, under the writer's direction, of eight representative, general-purpose warehouses in New York, N. Y., Philadelphia, Pa., Baltimore, Md., and Washington, D. C. Two buildings were surveyed in each city. The buildings varied in height between six and eleven stories.

Method of Conducting Surveys.—An engineer visited each building and selected from among the columns the one that, for its full height, had the largest percentage of its entire tributary area most heavily loaded. The rated or official live load, the actual live load on each floor and the tributary area were obtained.

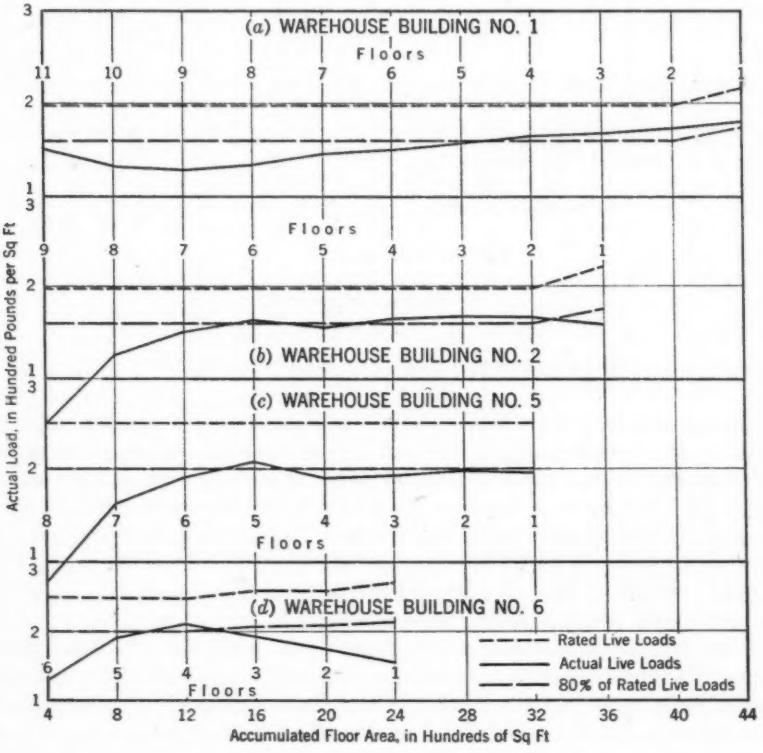
(a) WAREHOUSE BUILDING NO. 1
Floors
(b) WAREHOUSE BUILDING NO. 2
(c) WAREHOUSE BUILDING NO. 5
(d) WAREHOUSE BUILDING NO. 6
Rated Live Loads
Actual Live Loads
80% of Rated Live Loads
Actual Load, in Hundred Pounds per Sq Ft
Accumulated Floor Area, in Hundreds of Sq Ft

FIG. 5.—LIVE LOADS ON WAREHOUSE COLUMNS

This information for the four cases in which the actual live load was highest relative to the rated live load has been plotted in Fig. 5. The abscissas are cumulative tributary areas by floors. The average, actual live loads corresponding to each accumulation of area are indicated in each case by the full lines. The average, rated live loads corresponding to each accumulation of area are indicated by the dotted lines. For instance, the sixth story of the column in building No. 2, Fig. 5(*b*), was carrying a tributary area of 1,200 sq ft. The average, actual live load upon it was 150 lb per sq ft and the average, rated live load for that area is 200 lb per sq ft.

The most heavily loaded stories measured from the top down are summarized in Fig. 6. The heavy line in this figure indicates for each story the heaviest average, actual live load expressed as a percentage of the average,

rated live load. For instance, the maximum ratio of average, actual live load to average, rated live load on the story below the top floor in any of the buildings was 0.75 and occurred on the column surveyed in warehouse building No. 1 (Fig. 5(*a*)).

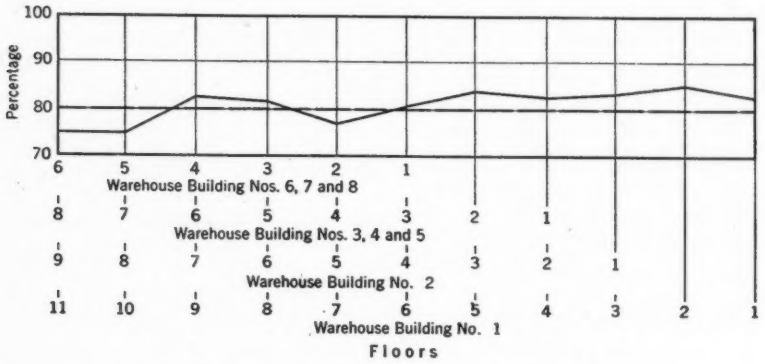
100
90
80
70
Percentage
6 5 4 3 2 1
Warehouse Building Nos. 6, 7 and 8
8 7 6 5 4 3 2 1
Warehouse Building Nos. 3, 4 and 5
9 8 7 6 5 4 3 2 1
Warehouse Building No. 2
11 10 9 8 7 6 5 4 3 2 1
Warehouse Building No. 1
Floors

FIG. 6.—SUMMARY OF THE MOST HEAVILY LOADED STORIES

Significance of Survey Data.—The data show that, contrary to the trend found in office buildings, there is a tendency on the most heavily loaded column of storage buildings for the average, actual live load to increase as the accumulation of tributary areas increases. This arises from a general practice of placing comparatively light loads on the top floors. The data also indicate that rarely is any story loaded with an average, actual live load of more than 80% of the average, rated live load. A dashed line has been added to each part of Fig. 5 indicating 80% of the average, rated live load.

Basic Live Loads and Design Live Loads.—As in the case of office buildings it appears that the basic live load for storage buildings should be the maximum average floor load that it is desired to permit in any bay. Furthermore, the basic live load should be used unreduced for the floor and beam design. In the light of the data presented herein the basic live load for storage buildings should be reduced for storage buildings as for office buildings with a maximum reduction of 20% for the design of columns. In the columns surveyed, such a reduction would result in a maximum excess of average, actual live load over design live load of 6¼%. The percentage overstress resulting from this condition would be less than 6¼% depending upon the weight of the construction.

6. CONCLUSIONS

The foregoing facts and reasoning seem to the writer to justify the conclusion that it is sound to derive design live loads from basic live loads as follows:

A. The basic live load shall be the maximum average live load which it is intended shall be allowed on any panel of a floor of a building.

B. When the basic live load is 100 lb per sq ft or less, for any occupancy except assembly occupancy, the basic live load may be reduced for the design of

any member at the rate of 0.08% per sq ft of tributary area; but the maximum reduction shall not exceed Δw_L as determined by Eq. 1.

C. In assembly occupancies the basic live load shall not be reduced for the design of any member.

D. When the basic live load is more than 100 lb per sq ft, it shall not be reduced for the design of floors, beams, girders, or trusses. It may be reduced at the rate of 0.08% per sq ft of tributary area for the design of columns but the maximum reduction shall not exceed 20%.

7. Acknowledgment

The field work in connection with the surveys of the Internal Revenue Building and the Veterans Administration Building was performed by P. H. Heimer and J. W. McLure, and by J. H. Feehan of the Office of the Supervising Engineer, U. S. Public Buildings Administration. The field work in connection with the surveys of the warehouses was performed by H. R. Williams of the Office of the Supervising Architect, U. S. Public Buildings Administration. Mr. Williams also prepared the figures and tables and reviewed the text of the original report. For permission to publish the survey data the writer is indebted to W. E. Reynolds, M. ASCE, Commissioner of Public Buildings.

For their cooperation and helpful suggestions in connection with the surveys of warehouses, thanks are also due to Warren T. Justice, president, The Pennsylvania Warehouse and Safe Deposit Company, Philadelphia, Pa., and to the following members of the Merchandise Division, American Warehousemen's Association: Charles E. Nichols, M. ASCE, manager, Washington, D. C.; F. T. Leahy, executive vice-president, Port of New York, New York, N. Y.; L. E. Naumann, president, and F. L. Malsey, vice-president, Baker and Williams, New York; Thomas E. Witters, president and general manager, Baltimore Fidelity Warehouse Company, Baltimore, Md.; W. E. Edgar, superintendent, Terminal Storage Company, Washington, D. C.; and J. P. Johnson, vice-president, Terminal Refrigeration and Warehouse Corporation, Washington, D. C.

The original manuscript from which this paper was prepared, containing unpublished tabular data, has been placed on file for reference at the Engineering Societies Library.[4a]

[4a] 33 W. 39th St., New York 18, N. Y.

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PAPERS

EFFECT OF STRESS DISTRIBUTION ON YIELD POINTS

BY F. G. ERIC PETERSON,[1] ASSOC. M. ASCE

SYNOPSIS

In practice, the yield point of mild and structural steel is obtained by a standard axial tension test. The stress-strain diagram for compression has been found to agree closely with that for tension. After the yield point has been obtained by this standard tension test it is commonly assumed that when any fiber in a steel structure reaches such stress intensity, yielding will occur. Such is not the case; yielding does not occur until higher stress intensities are reached when the distribution of stress over the critical section is not uniform. The stress at which yielding occurs is dependent on type of stress distribution and the shape of the cross section under stress.

The application of loads on a beam to cause bending, and eccentric tension tests, are two of the simplest ways in which to produce a nonuniform stress distribution. Both of these methods have been used in obtaining the data for this paper.

PART I. THEORIES OF PLASTIC FAILURE IN BENDING

Two theories of ultimate moment resistance, M_u, of a given cross section have been developed (1).[2] These will be referred to as the "old theory" and the "new theory." Both theories, no doubt, idealize the actual conditions that exist.

The "Old Theory."—This theory is based on an idealized stress-strain diagram (work line) as shown in Fig. 1(*a*) by the broken line OAB which is substituted for the curved line and is a very close approximation to the curved line. Fig. 1(*b*) shows the stress distribution over a given cross section in pure bending with the extreme fiber stress σ_y below the yield point. Fig. 1(*c*) shows the stress distribution after some of the outer fibers have exceeded the

NOTE.—Written comments are invited for immediate publication; to insure publication the last discussion should be submitted by September 1, 1946.

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[2] Numerals in parentheses, thus: (1), refer to corresponding items in the Bibliography (see Appendix).

yield point and, on the basis of the idealized stress-strain relation, would be as shown in Fig. 1(*d*) where all moment resistance M_u is practically exhausted.

This theory assumes that the ultimate moment resistance is reached when all fibers (both tension and compression) have reached the lower yield point as determined by the usual tension test. It can be seen that to have fibers

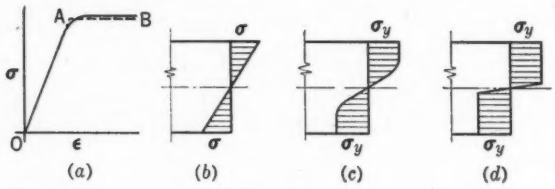

FIG. 1

very near to the neutral axis reaching the yield point the necessary strain requires considerable curvature of the member at the cross section and theoretically connot actually hold right up to the neutral axis. Nevertheless, this conception of yielding gives values that are good approximations to the actual since fibers very near to the neutral axis contribute very little to the moment resistance in any event. When a section has reached the ultimate value of M_u it is said to act as a yield hinge. The action of such a hinge is not reversible since the behavior on reversing becomes elastic. In the case of I-beams with the web vertical, failure is likely to occur before the "yield-hinge" condition is reached because the compression flange will usually buckle soon after this entire flange has reached the yield point.

The "New Theory."—This theory is based on the idea that the yield point as obtained by the usual tension test does not necessarily hold for cases of nonuniform stress distribution. In the simple tension test of structural steel the yield point is determined by the occurrence of one or more phenomena. These are: (*a*) The drop of the scale beam of the testing machine, (*b*) the scaling off of the surface material in hot-rolled steel, and (*c*) the appearance of Lueders lines or yield lines in the case of a polished specimen. The appearance of Lueders lines in the polished specimen is without doubt the same phenomenon that causes the scaling of the surface in the hot-rolled specimen. In polished specimens which are subjected to nonuniform stress conditions the forming of Lueders lines occurs at a higher stress than in the simple tension test. From tests by A. Thum and F. Wunderlich (2) of pure bending of various forms of cross sections, the stress at which Lueders lines appear is dependent on the shape of the cross section. More particularly it seems to be dependent on whether the greater part of the material is distributed close to the neutral axis (as in the case of a round bar) or most of the material is distant from the neutral axis (as in the case of an I-beam or wide flange section). In the first case the yield point in bending appears to be raised a considerable amount (about 65% in the case of a round bar (3)) whereas in the second case the increase is not so marked. In the case of a rectangular cross section the increase in the yield

point is somewhat more than 40%. Tests by the writer (detailed report of which immediately follows this discussion) seem to bear out these contentions. In the case of pure bending of two polished rectangular specimens the writer found that the stress at which Lueders lines formed was, respectively, 40.5% and 44.0% higher than in the simple tension test, for an average value of about 42.2%. In these tests readings of deflection of the beam at the midpoint were also taken. It is interesting to note (shown subsequently in Fig. 5) that the load-deflection line appears to continue to be a straight line practically up to the appearance of the Lueders lines. This tends to confirm the idea that the yield point is actually raised since otherwise the load-deflection line should tend toward the horizontal at points where the extreme fiber stress is higher than the yield point obtained from the simple tension test.

Further tests by the writer, as reported in Part II, were made on four specimens in eccentric tension to determine whether similar results would be obtained, and also to determine whether the ratio of thickness to width had any bearing on the results. These specimens were also of rectangular cross section. In all four cases the apparent raising of the yield point was more than 40% and the ratio of thickness to width seemed to have little effect. In the thinner specimens the Lueders lines or yield lines formed at right angles to the load, indicating yield across the thickness of the specimen governed. In the thickest specimen, lines were formed perpendicular to the load and also at 45° to the load indicating yield across the thickness and the width of the specimen simultaneously. It is well known that these lines form along the directions of maximum shear stress.

In the tests by Messrs. Thum and Wunderlich (2) a tension test on part of a bar of steel showed it to have a yield point of 38,400 lb per sq in. Next a rectangular specimen was cut from the same bar, polished, and then subjected to bending. A load which, by the usual bending formula, would cause an extreme fiber stress of 49,000 lb per sq in., or 27.5% greater than the tension yield point, was applied 200 times consecutively without the appearance of yield lines or any other sign of failure.

This "new theory" contends that in the case of nonuniform stress the yield point is higher than in the uniform state as indicated by the usual tension test. It also states that the increase $\Delta\sigma_y$ over the tension yield point σ_y is dependent on the shape of the cross section and that when this increased yield point is reached yield occurs suddenly and that the stress then subsides to the tension yield point. This phenomenon has been likened to the case of the delayed boiling of a liquid (4). The explanation of the phenomenon possibly lies in the field of metallography. A. M. Freudenthal, Assoc. M. ASCE (4), states that it is confined to metals that have a "stereocentric" type of lattice structure and is not evident in the "planocentric" or more simple types of lattice structure. The phenomenon of the raising of the yield point, no doubt, is confined to metals that have a very well-defined yield point and therefore includes mild steel and structural steel.

With the knowledge that is available at present it is probably impossible to give a rigid mathematical treatment of the phenomenon. A basic difficulty presents itself in the fact that all the investigator can measure is strain—that

is, he cannot measure stress directly. The difficulty from the mathematical viewpoint is that although much is known about elastic behavior, and also about completely plastic behavior, little knowledge is available concerning the range between these states where plastic regions are surrounded by elastic material, and elastic portions are surrounded by plastic regions.

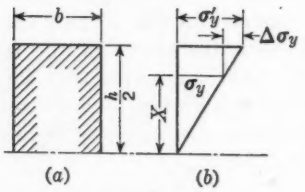

FIG. 2

Nevertheless, W. Kuntze (3) has developed a method of determining the apparent increase in the yield point which agrees well with the results of numerous tests performed. This method can be stated as follows: Assuming that the triangular distribution of stress in bending holds, even if the tension yield point is exceeded, divide the total stress on one side of the neutral axis into two equal parts; then set the stress at the point which divides these two parts equal to the tension yield-point stress.

For illustration, assume that a rectangular bar is subjected to bending, neglecting the shearing stresses. Fig. 2 shows the upper half of a rectangular cross section in bending, in which X is the distance from the neutral axis to the point that divides the total stress into two equal parts. Then

$$X = \tfrac{1}{2}\, h\, \frac{\sigma_y}{\sigma'_y} \qquad (1a)$$

and

$$\frac{\sigma'_y}{2} \times \frac{b\,h}{2} = b\,X\,\frac{2\,\sigma_y}{2} \qquad (1b)$$

Eliminating X from Eqs. 1: $\sigma'_y = \sqrt{2}\,\sigma_y$; or $\Delta\sigma_y = 0.414\,\sigma_y$, which represents a 41.4% increase. It is seen that this value agrees well with the results obtained by experiment. In a series of eight beam tests by Messrs. Thum and Wunderlich (2) an average value of 40.5% increase was obtained. The tests by the writer in both bending and eccentric tension resulted in an average increase of 42.4%, test I in eccentric tension (see Part II) was omitted from this average. Fig. 3, a plat of data by Mr. Kuntze (3), affords a graphic comparison of the values of X obtained by computation and experiment for various types of cross sections. The correlation is very good.

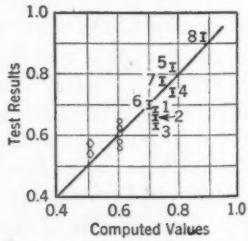

FIG. 3.—COMPARISON OF COMPUTED AND EXPERIMENTAL VALUES OF $X = \frac{\text{TENSION YIELD POINT}}{\text{BENDING YIELD POINT}}$

Calculation of the ultimate bending-moment resistance, M_u, on the basis of the "new theory" gives values slightly less than those given by the "old theory"; and they agree better with the results of tests as will be seen in a study made by Mr. Kuntze (3). Whether or not the "new theory" is actually correct, calculation on this basis is simplified since, after σ_y is found, the usual section modulus can be used in calculating M_u, that is

$$M_u = \sigma_y\, Z \qquad (2)$$

An account of the results obtained by the writer in some tests on bending and eccentric tension is presented in Part II.

PART II. TESTS RESULTS ON THE APPARENT RAISING OF THE YIELD POINT OF STEEL UNDER NONUNIFORM CONDITIONS OF STRESS

1. Pure Bending

These tests were made to determine inconsistencies in results by the usual tension test and those obtained under nonuniform stress. One specimen was tested in pure tension and two specimens in pure bending. Fig. 4 shows the dimensions and the arrangement of the supports and points of application of

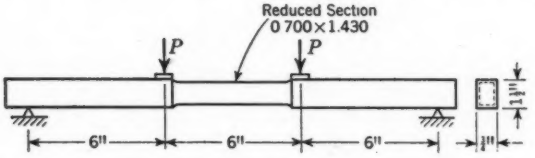

Fig. 4

the loads for the bending tests. The specimens were of cold-rolled flat bar steel $1\frac{1}{2}$ in. by $\frac{3}{4}$ in., machined to 0.700 in. by 1.430 in. at the narrow section. Consequently, $A = 1.002$ sq in. and $Z = 0.239$ in.^{3} A similar specimen was prepared for the standard tension test. All three specimens were annealed at 1,700° F and allowed to cool slowly in the furnace, after which the narrow section was polished so as to facilitate the observation of Lueders lines. Since 1,700° is well above the normalizing temperature the internal stresses due to cold rolling were eliminated. All three specimens were cut from the same bar.

The tension-test specimen showed an upper yield-point stress of 36,080 lb per sq in. and a lower yield point of 35,370 lb per sq in. The Lueders lines appeared immediately after the upper yield point was reached and spread quite rapidly over the entire specimen.

TABLE 1.—Percentage Increase in Yield Point, by Bending Test

Col. 1, specimens 1, 2, and their average.
Col. 2, Lueders line load.
Col. 3, unit stress σ in kips per square inch.
Col. 4, percentage increase beyond the tensile yield point.

Bar (1)	Load (2)	σ (3)	$\Delta\sigma_y$ (%) (4)
1	3,960	49.7	40.5
2	4,045	50.9	44.0
Average	(4,002)	(50.3)	(42.2)

The specimens tested in bending were subjected to equal loads P at the third points so that the middle section was in pure bending. If the yield point in pure tension had governed the appearance of yielding in bending as evidenced by the formation of Lueders lines, these lines should have appeared at a load of 2,820 lb. Such was not the case, however. Table 1 shows the results obtained. According to the method described by Mr. Kuntze for pure bending of a rectangular cross section, the increase in the yield point should be $\Delta\sigma_y = (\sqrt{2} - 1)\sigma_y$ or about 41.4%. Considering experimental error the results of these tests agree very well with this value.

As was to be expected, the Lueders lines appeared first on the upper and lower surfaces of the cross section in pure bending. They appeared almost simultaneously on these surfaces and spread quite rapidly down the sides of the specimen from the top surface, and up the sides of the specimen from the lower surface, with very little increase in the load. They reached a point

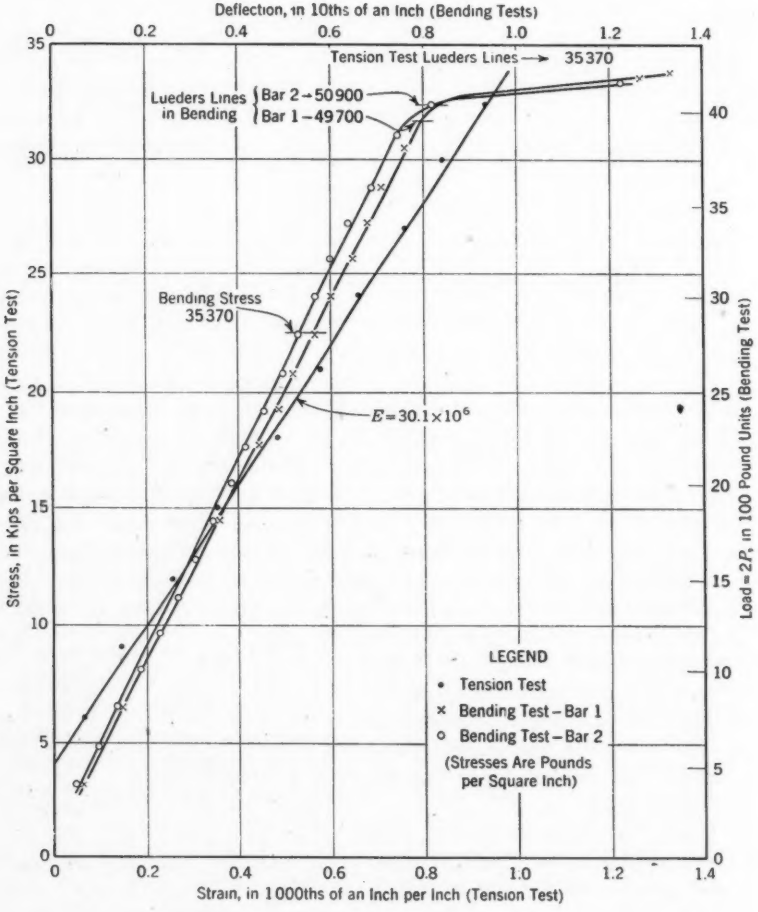
Deflection, in 10ths of an Inch (Bending Tests)
Tension Test Lueders Lines → 35370
Lueders Lines in Bending
Bar 2 – 50900
Bar 1 – 49700
Bending Stress 35370
$E = 30.1 \times 10^6$
LEGEND
Tension Test
Bending Test – Bar 1
Bending Test – Bar 2
(Stresses Are Pounds per Square Inch)
Stress, in Kips per Square Inch (Tension Test)
Load = 2P, in 100 Pound Units (Bending Test)
Strain, in 1000ths of an Inch per Inch (Tension Test)

FIG. 5

about $\frac{1}{4}$ in. from the neutral axis each way. This seems to indicate that the process of yield in bending is a fairly rapid phenomenon rather than a slowly expanding region of yield.

The results of these tests are shown graphically in Fig. 5. Note that the load-deflection line appears to continue as a straight line for bending stresses far above the tension yield-point stress of 35,370 lb per sq in. Note also that

after Lueders lines have appeared the load-deflection lines tend to become horizontal, and the bending resistance is practically used up.

2. Eccentric Tension Tests

These tests were made in order to determine the effect of the nonuniform stress condition in eccentric tension on the apparent raising of the yield point

TABLE 2.—Dimensions of Specimens, Eccentric Tension Tests

Specimen	Thickness b, in Inches		Width h, in Inches		Area A, in Square Inches		Section modulus Z (in.^{3})
	Axial	Eccentric	Axial	Eccentric	Axial	Eccentric	
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
I	0.048	0.048	0.627	1.800	0.0301	0.0855	0.0257
II	0.1115	0.111	0.627	1.800	0.0698	0.200	0.060
III	0.234	0.233	0.628	1.800	0.147	0.429	0.126
IV	0.351	0.350	0.626	1.800	0.220	0.630	0.189

as compared to the yield point determined by axial tension. Four specimens of different thicknesses were used to discover whether the ratio of thickness to width had any effect on the results. Control tests in axial tension were made for each specimen. The steel used in these control tests was obtained from the same bars of steel, respectively, from which the eccentric specimens were obtained.

The metal for these tests was selected at random from a commercial stock. Therefore, there was no reason to expect the yield point of the individual specimens to agree very closely. The steel was hot rolled. The dimensions of each specimen are presented in Table 2. The form and dimensions of the eccentric tension specimen are illustrated in Fig. 6. The eccentricity was one sixth of the width so that the stress varied from zero at one edge to twice the average at the other edge. Significant phenomena observed at various loads (in pounds) are as follows:

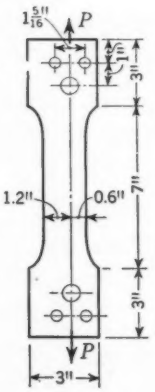

Fig. 6

Load (lb)	Remarks
Specimen I, Axial.—	
1,300	Lueders lines formed near the end curves but did not spread.
1,500	Lines appeared near the center of the specimen and spread over the entire narrow section.
1,600	This was the maximum load applied.
Specimen I, Eccentric.—	
3,300	Lines first appeared at the stressed edge and spread over the length of the specimen. As the load increased, the lines gradually extended laterally toward the zero stress edge.
4,000	This was the maximum load applied.

Load (lb)	Remarks
Specimen II, Axial.—	
2,610	Upper yield point. Lueders lines began to form.
2,590	Lower yield point.
3,000	This was the maximum load applied.
Specimen II, Eccentric.—	
4,700	Lueders lines began to form near the curves at the ends of the reduced section.
5,400	Lueders lines began to form near the middle of the reduced section, at the stressed edge.
6,540	This was the maximum load applied. The formation of Lueders lines near the curved ends, under the 4,700-lb load, must have been caused by stress concentration because, after they formed, no tendency to spread was noticed until lines began to appear near the middle section of the specimen (the 5,400-lb load). Even at these points the increase was 26.5%. At the first appearance of Lueders lines they extend ½ in. or more laterally.
Specimen III, Axial.—	
6,340	Upper yield point. Lueders lines began to form.
6,240	Lower yield point.
6,550	This was the maximum load applied.
Specimen III, Eccentric.—	
12,850	Lueders lines began to form near the middle of the reduced section.
15,000	This was the maximum load applied.
Specimen IV, Axial.—	
10,750	Upper yield point. Lueders lines began to form.
9,900	Lower yield point.
12,000	This was the maximum load applied.
Specimen IV, Eccentric.—	
18,250	Lueders lines began to form at the ends of the reduced section—that is, where the end curves begin.
20,000	Lines began to form near the middle of the narrow section at the stressed edge and spread over the entire specimen.
22,500	This was the maximum load applied.

From the foregoing data it was possible to compute the percentage increase $\Delta\sigma'_y$ as shown in Table 3.

The results of these tests indicate an appreciable increase in the yield point (assuming that Lueders lines indicate the yield point) in the nonuniform state of stress as compared with the yield point resulting from axial tension; they agree quite well with the results obtained in pure bending. Each of the specimens showed an increase of more than 40% and the thinner specimens seemed to show the greatest increase. This was contrary to what was expected as it was thought that, when the axial tension yield point was reached across the thickness of the section at one edge, the Lueders lines would appear. Fig. 7

TABLE 3.—INCREASE IN YIELD-POINT STRESS BY BENDING

Specimen No.	CRITICAL LOAD (POUNDS)[a]		STRESS (LB PER SQ IN.)		Increase, $\Delta\sigma'_y$, Col. 5 over Col. 4 (%)
	Axial	Eccentric	Axial[b] σ_y	Eccentric[c] σ'_y	
(1)	(2)	(3)	(4)	(5)	(6)
I	1,500	3,300	49,900	77,200	55
II	2,590	5,400	37,100	54,000	45.5
III	6,240	12,850	42,450	60,000	41.3
IV	9,900	20,000	45,000	63,500	41.0

[a] See tabulation in the text, and accompanying remarks. [b] $\frac{\text{Col. 2}}{\text{Col. 6, Table 2}}$. [c] $\frac{2 \times \text{Col. 3}}{\text{Col. 7, Table 2}}$.

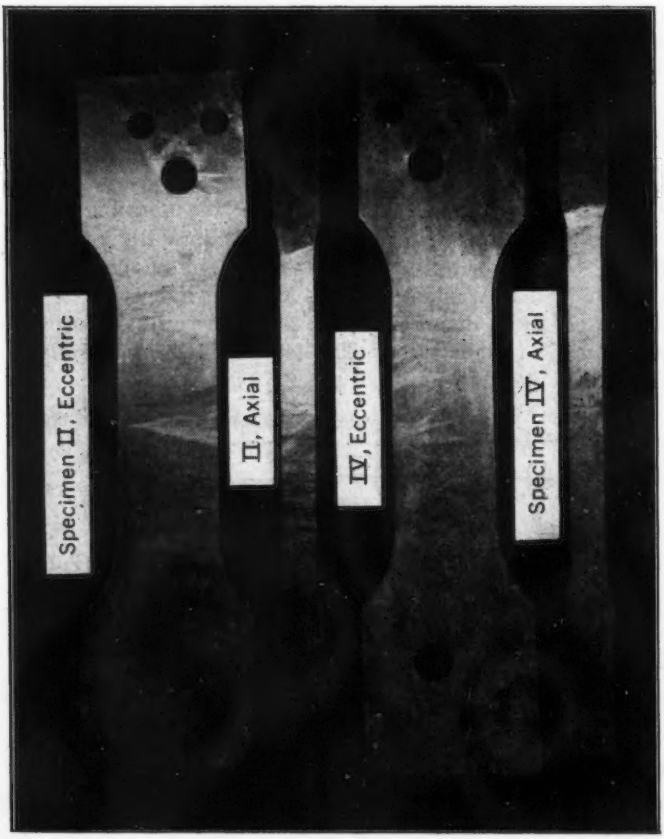

FIG. 7

is a photograph showing the Lueders lines on specimens II and IV, for axial and for eccentric tests. On specimens II, which are quite thin, note that the lines are predominantly horizontal whereas on specimens IV, which are relatively thick, there is a marked tendency of the lines to spread at 45° as well. This is apparently the only effect of thickness. If it is assumed that there is a straight line stress distribution up to the appearance of Lueders lines the apparent raising of the yield point is just as marked for the thin as for the thick specimens. On the three thinnest specimens the Lueders lines formed at right angles to the direction of the load although after reaching a considerable depth there was a tendency to form at 45°. On the thickest specimen the lines formed both at 90° and at 45° to the direction of the load. This seemed to be the only effect of the thickness-to-width ratio. As was shown by Messrs. Thum and Wunderlich (2) and by Mr. Kuntze (3) the shape of the cross section has a marked effect on the raising of the yield point.

The appearance of Lueders lines seems to be the first indication of physical breakdown of the internal structure of the metal. It may be open to question whether or not yield occurs before Lueders lines begin to form. Does the metal reach the plastic state in advance of the appearance of Lueders lines? Referring to Fig. 5, the load-deflection lines of bars 1 and 2 appear to continue in the same straight line far beyond the point at which the stress in the extreme fiber has reached the tension yield point. If this is true, the beam continues to react elastically even after the tension yield point has been reached.

Check Tests.—As a check on the reliability of the foregoing tests, in support of the conclusion that the yield point is actually raised, the writer decided to make further tests in which Lueders lines would not be observed. The writer does not contend that Lueders lines are an exact indication of yield. Since the tension stress-strain diagram shows some curvature at a stress somewhat lower than the yield point (as usually defined), some slight yielding occurs below the yield point and before Lueders lines appear.

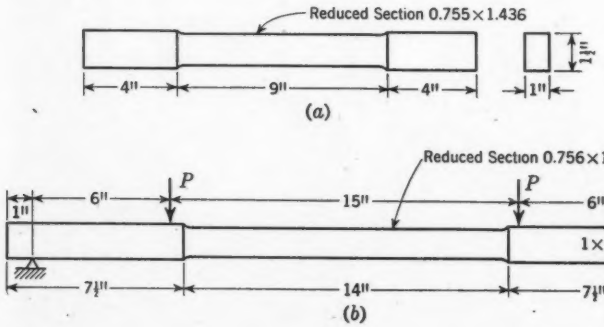

FIG. 8

Two rectangular specimens were tested in tension and two in bending. The dimensions of these specimens are shown in Fig. 8. All specimens were cut from the same bar of commercial mild steel. Fig. 8(*a*) gives the dimensions of the two tension specimens. Fig. 8(*b*) gives the dimensions of the bending

specimens and also the arrangement of loading. This loading arrangement causes the middle 15-in. section to be subjected to pure bending. By means of the frame shown in Fig. 9, which was attached to the reduced section of the bending specimen, it was possible to read, accurately, the deflection at the mid-

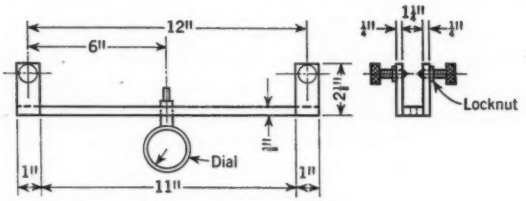

FIG. 9.—DIAL GAGE HOLDER

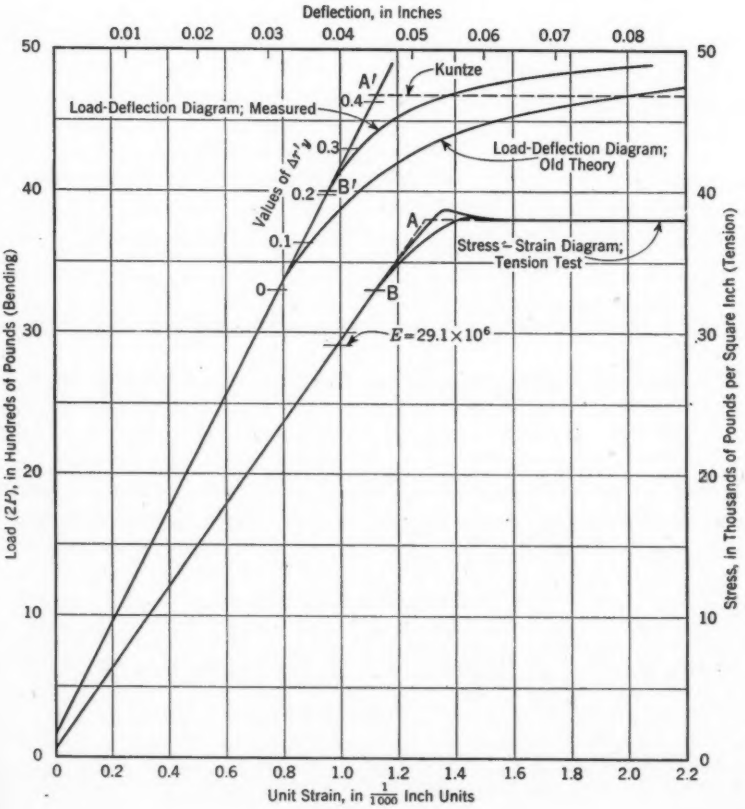

FIG. 10

point of a 12-in. section (subjected to pure bending moment) relative to the ends of that section. A dial, reading directly to 0.0001 in., was used.

The results of these tests are shown graphically in Fig. 10. The stress-strain diagrams for the tension specimens were identical except near the yield

point where two distinct curved lines appear. The yield points from direct calculation of load over area were, respectively, 37,960 lb per sq in. and 38,050 lb per sq in. The difference between these two values is not sufficient to show on the graph. The dotted lines intersecting at point A represent the idealized stress-strain diagram later assumed in calculations. In Fig. 10 is also shown the measured load-deflection diagram for the two bending specimens. The deflections for each specimen were so similar that only one curve appears.

The question now to be considered is whether or not this measured load-deflection diagram conforms with the tension stress-strain diagram (assuming that this diagram also holds for direct compression). Assume also that a plane section before bending remains plane after bending. Since no direct shear is involved this assumption is quite justified.

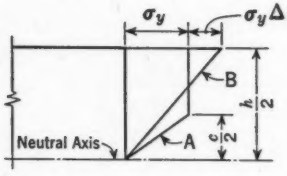

FIG. 11

If a load is applied which is larger than necessary to bring the extreme fibers to the tension yield-point stress, the stress diagram for pure bending must be either as diagram A, Fig. 11, which is based on the idealized tension stress-strain diagram, or as diagram B which assumes an increase of the yield point. In either case the moment of the stresses about the neutral axis for a given load must be the same. The moment about the neutral axis according to diagram A, Fig. 11, is

$$M_1 = 2\left[b\left(\frac{h}{2} - \frac{c}{2}\right)\tfrac{1}{2}\left(\frac{h}{2} + \frac{c}{2}\right) + \tfrac{1}{2}\, b\, \frac{c}{2}\,\frac{2}{3}\,\frac{c}{2}\right]\sigma_y = \frac{b}{4}\left(h^2 - \frac{1}{3}\,c^2\right)\sigma_y \quad \ldots (3a)$$

The moment about the neutral axis according to diagram B, Fig. 11, is

$$M_2 = (1 + \Delta\sigma_y)\,\sigma_y\, Z = (1 + \Delta\sigma_y)\,\sigma_y\,\frac{b\,h^2}{6} \quad \ldots (3b)$$

in which Z is the section modulus and $\Delta\sigma_y$ is a factor indicating how much the yield point is exceeded (expressed as a ratio). Setting these moments equal to each other, $c = h\sqrt{1 - 2\,\Delta\sigma_y}$; or

$$c/2 = h/2\,\sqrt{1 - 2\,\Delta\sigma_y} \quad \ldots (4)$$

Because the moment is constant over the part of the beam considered, the curvature is circular. If diagram A, Fig. 11, governs, the radius of curvature is

$$\rho_1 = \frac{0.5\,c}{\epsilon} = \frac{c\,E}{2\,\sigma_y} \quad \ldots (5a)$$

in which ϵ is the unit strain at a distance $c/2$ from the neutral axis. If diagram B governs,

$$\rho_2 = \frac{E\,I}{M_2} = \frac{E\,I}{(1 + \Delta\sigma_y)\,\sigma_y\, Z} = \frac{E\,h}{2\,(1 + \Delta\sigma_y)\,\sigma_y} \quad \ldots (5b)$$

and

$$\frac{\rho_1}{\rho_2} = \frac{h}{(1 + \Delta\sigma_y)\,c} = \frac{1}{(1 + \Delta\sigma_y)\,\sqrt{1 - 2\,\Delta\sigma_y}} \quad \ldots (6)$$

using Eq. 4. Because of the large radius of curvature as compared with the length of the specimen involved, the ratio of the deflections is inversely proportionate to the radii of curvature, that is

$$\frac{\delta_1}{\delta_2} = \frac{1}{(1 + \Delta\sigma_y)\sqrt{1 - 2\Delta\sigma_y}} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (7)$$

in which δ_1 is the midpoint deflection according to diagram A and δ_2 is the same for diagram B. For as far as diagram B governs, the load-deflection line will be a straight line; but, if diagram A governs, the load-deflection line will curve toward the horizontal as the load is increased after the tension yield point has been reached in the extreme fibers.

Referring to Fig. 10, the load in bending at which the extreme fibers reach the yield point in tension is 3,300 lb as indicated by the line $\Delta\sigma_y = 0$. If diagram A, Fig. 11, governed, the load-deflection line would cease to be a straight line above this value and would diverge from the straight line for various values of $\Delta\sigma_y$ according to Eq. 7. Values of the load and the ratio of δ_1 to δ_2 are given in Table 4 for various values of $\Delta\sigma_y$. The curve resulting

TABLE 4.—VARIATION OF δ_1/δ_2

Description	VALUES OF $\Delta\sigma_y$										
	0.00	0.05	0.10	0.15	0.20	0.25	0.30	0.35	0.40	0.45	0.50
Load	3,308	3,480	3,640	3,805	3,976	4,140	4,300	4,465	4,630	4,800	4,962
δ_1/δ_2	1.000	1.004	1.016	1.040	1.077	1.132	1.218	1.350	1.597	2.180	∞

from these tabulated values is shown in Fig. 10 marked "Load-Deflection Diagram; Old Theory." The load-deflection diagram according to Mr. Kuntze (3) is shown by the dotted line marked "Kuntze." The Kuntze "new theory" assumes a sudden yielding at a unit stress 41.4% higher than the tension yield point, for a rectangular section.

Now consider the load-deflection diagram (measured). It continues as a straight line up to a load of 4,000 lb, point B′, Fig. 10, giving an extreme fiber stress of 47,500 lb per sq in. This should be defined as the proportional limit in bending. Note that it is 25% higher than the tension yield point. Actually, this value of 47,500 lb per sq in. should be compared with the tension proportional limit. Realizing that this value is never very definite it still does not seem amiss to make an extimate of its value. Referring to the tension stress-strain diagram, a value of 33,000 lb per sq in. (point B, Fig. 10) seems about right. Then the ratio of the proportional limit in bending to the proportional limit in tension would be 47,500/33,000 = 1.44, or an increase of 44%.

For values of the load in bending greater than 4,000 lb the measured load-deflection diagram diverges from the straight line but is much closer to the straight line than to the load-deflection curve based on the "old theory." For a value of $\Delta\sigma_y = 0.30$ the divergence from the straight line according to Table 4 should be 21.8%. It is only 3%. For $\Delta\sigma_y = 0.40$ it should be 59.7% whereas it is actually only 14.5%.

Since the load-deflection diagram of Mr. Kuntze is based on the idealized stress-strain diagram, it was to be expected that the load deflection (measured) should diverge from the straight line at some point below A′ (Fig. 10) in order to conform with the fact that the actual stress-strain diagram begins to curve somewhat below point A.

Thus these latter tests, made without reference to Lueders lines, tend to confirm the fact of an increase in the yield point over that determined by the ordinary axial tension test when the stress is not uniformly distributed.

Conclusions

The data presented in this paper show that the ordinary axial tension test of mild and structural steel does not give information as to the behavior of the material under nonuniform stress conditions. The stress at which yield occurs is higher under nonuniform stress and is dependent on the kind of stress distribution and also on the form of the cross section under stress.

This fact tends to minimize the importance of stress concentrations since the stress distribution over an entire cross section, rather than the stress at an isolated point must be considered in order to determine when yielding will occur. Mr. Kuntze (3) reports that in tension tests of a flat bar with a round hole through it, the stress near the hole may reach a value from 2.3 to 2.7 times the tension yield-point stress before there is evidence of yield.

Failure due to fatigue stresses is not considered in this paper and, therefore, the paper is confined to cases of static loads or cases where the number of repetitions of loading is relatively small. Nevertheless, in tests by K. Kloppel (5) in the loading of a beam to the tension yield point in the extreme fibers, the load was repeated 700,000 times "without showing any signs of premature breakage through fatigue," and "the permanent deflections were practically zero."

APPENDIX

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(18) **"Beitrag zum Traglastverfahren,"** by F. Stüssi and C. F. Kollbrunner, *Die Bautechnik*, Vol. 13, 1935, p. 264.

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AMERICAN SOCIETY OF CIVIL ENGINEERS

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PAPERS

SOME THOUGHTS ON ENGINEERING EDUCATION

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SYNOPSIS

The views of a practicing engineer on engineering education as provided by engineering schools and colleges in the prewar period are presented in this paper; and some suggestions are offered for changes and modifications in academic courses for the postwar era. Supported by appropriate data, the paper discusses the organization of the engineering profession, and the general conditions encountered in the practice of engineering. It argues in support of the conclusions that postwar engineering education should emphasize fundamental principles in subjects throughout the broad field of engineering; and that postwar engineering education should also include courses to prepare a graduate for a career involving administrative, executive, and managerial activity, both technical and nontechnical in character. This paper holds that specialization, or the acquisition of knowledge of the advanced theories and their application to design, operation, and processes, should be primarily left to the years after graduation and that engineering schools should provide such positive and formal guidance and assistance to their alumni in this postgraduate period as will encourage and even urge them to pursue systematic studies toward such specialization.

The paper also urges that engineering educational institutions and engineering societies conduct research concerning the engineering profession, the practice of engineering, and the market for engineering services, in the postwar period—in other words, that engineers as well as engineering be given adequate attention.

INTRODUCTION

Engineering education, as offered by schools and colleges for the several decades prior to World War II, has kept pace with changes and advances that

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have occurred in technology, through modification of existing course content and by the addition of new courses to the curriculum. It has not, however, made comparable progress in keeping abreast of changed conditions that have developed during that period in the pattern and structure of the engineering profession, in the practice of engineering, or in the market for engineering services.

Prewar engineering courses (at least in the opinion of graduates, if not from the viewpoint of subject content) tended more and more toward specialization in one of the major branches of engineering, or even in one of the subspecialities of engineering. Emphasis was laid on depth rather than on breadth of instruction, and on imparting skills and knowledge of practice, when experience shows a strong probability that the graduate through changing employment conditions would encounter little opportunity to utilize such skills, or would not have the responsibility for directing such practice, for a long period following his graduation. Little provision was made for furnishing directed guidance, organized assistance, or specific encouragement to those students who, following graduation, desired to continue their studies along specialized lines or in other fields of engineering. The product of formal prewar engineering educational courses presents a picture of a trained rather than of an educated person.

The impact of the World War II on the educational system of the United States, and particularly on engineering education, has been such that in many schools prewar engineering courses practically ceased to exist. In many places, short courses in specialized subjects, and accelerated courses of longer duration designed to meet the needs of the armed forces, were substituted. With the end of the war, prewar courses are being reestablished.

Engineering faculties were disrupted during war years, most of their younger members being in uniform, whereas many older ones left, temporarily or permanently, for service to government or industry. Likewise, the source of supply of new instructors, normally drawn from the ranks of recent graduates, disappeared. As a result of the specialized training given many in the armed forces and in industry, however, much has been learned about the technique of instruction, and this knowledge will prove valuable in postwar courses.

It will require considerable time for engineering schools to readjust themselves to normal peacetime functioning. Many students who left prewar courses for the armed services, others who have attended short or accelerated courses, and many young men in service who may have enrolled in courses proposed to be given them during the interval between the end of hostilities and demobilization are seeking to return and secure degrees. These men are more mature and purposeful, and likewise more impatient to secure their degrees, than were the usual prewar upper classmen, and the problem of handling them will be somewhat difficult; but this situation will not last for more than a few years.

On the other hand, there are the usual crop of high school graduates seeking to enrol in normal postwar engineering courses. Once the general pattern and structure of postwar engineering education has become set, extreme difficulty will be encountered in making changes. If these latter are indicated, it is

time to discuss them, and they should be discussed not alone by engineering educators, but by practicing engineers and employers of engineers—laymen and professionals at all levels of responsibility—who constitute the consumers or users of the product in the market for engineering services.

This paper presents the views and opinions, not of an engineering educator, but of a practicing engineer who has been employed by engineers, and also has employed engineers, and who has had a rather unique opportunity to observe the engineering profession and the practice of engineering in an objective manner. The conclusions advanced are not susceptible of definite proof from available data, rather they are primarily matters of opinion based upon observation and contact with large numbers of engineers over a long period of time.

The major purpose of the paper is not so much to argue in support of such conclusions as it is to evoke discussion of the subject of engineering education by the users of its product.

It is not believed that prewar engineering education was a failure. Accomplishments of engineers during the past half century have proved its worth. Rather, it is believed that it has done a creditable job, but that a time has arrived when it should adjust itself to current and future conditions, and should endeavor to educate students not solely to become engineers, but readily to enter engineering or other fields where their education will be of equal value. Four years is too short a time for a student to acquire a knowledge of more than the basic fundamentals of engineering and their simple application. If he so desires, he should be afforded assistance and guidance in his further search for knowledge. Every engineering student must work hard to graduate. This probably was the most valuable single thing that he learned in his course.

Prewar Engineering Education

The general objective of most engineering courses offered during the fifteen to twenty years prior to World War II appeared too often to produce a graduate well versed in the theory of, and with an academic knowledge of, practice in one particular branch, and in many instances in one specialty in the field of engineering, thus equipping the graduate to enter such field or specialty on graduation and encouraging him to do so, irrespective of what the market for services in such field might be in five or ten years.

The standard length of course was four years. Attempts to lengthen it (as had been done in the case of preparatory courses in some of the other professions) have not met with any particular success.

Because of the magnitude and complexity of the field of engineering, and the limited time available for formal professional education, those responsible for the general design and planning of engineering courses have been faced with the following alternatives:

I. To develop a broad general course, in which the student was given a thorough foundation in the basic fundamentals underlying engineering; or

II. To develop a course in which less time was given to the basic fundamentals of alternative I and more time was devoted to advanced applications to, and practices in, a single branch or specialty, with limited instruction in nontechnical subjects.

A course such as alternative I would include mathematics and the physical sciences, with their application to engineering subjects such as simple structures, mechanics of materials, thermodynamics, electrical circuits, electrical machines, and hydraulics. In addition to instruction in these general, basic practices in the field of general engineering, the broad course would include instruction in nontechnical subjects that would be useful to the engineer in his subsequent career, whether that career led to engineering, commerce, or industry. The acquisition of particular knowledge in the various specialities would be left for the period following graduation.

In the prewar period too many engineering courses were the result of a choice of alternative II. During the first two years the student was instructed in mathematics through calculus, and was given one year each of chemistry and physics. He also had a certain amount of work in the shop, field, and drafting room, with possibly a course or so in one or more elemental specialized subjects and a few courses in nontechnical subjects. Beginning in his junior year, at the latest, courses dealt with the application of the subjects studied in lower division courses to the more specialized field that he had been required to select, with further instruction in theory and practice in such specialized field. He analyzed stresses and made designs of large bridges and dams, studied the economics of power plants or systems of high-tension transmission lines, or considered problems of layout or management of industrial plants. He grappled with problems of a character that he would never meet, or at least would not have to solve, on his own responsibility for from at least twenty to twenty-five years after graduation.

Degrees were usually granted in the major branch of engineering represented by the course followed as an undergraduate. If the student returned for postgraduate study, his course too often consisted of more intensive specialization or of research.

The result of this general pattern and structure of engineering education was that the graduate had a good theoretical knowledge of the field or specialty that he had followed, an academic knowledge of practice in this field or specialty, and a slight training in some of the skills that he might need in later years. Although he might be called upon to utilize these skills, in such activities as drafting, computing, use of machines, or use of precise instruments shortly after graduation, not until a very considerable period following that date would he have acquired sufficient maturity and experience to allow him to use independent judgment or to assume any substantial responsibility in the exercise of his knowledge of theory and practice.

As a graduate he seldom had the ability to express himself adequately or a knowledge of how to acquire this ability. He had little knowledge of business or of business principles or practices, of the behavior of people, or of human institutions; and he had little or no information as to where or how to acquire

this knowledge. He was primarily a trained, rather than an educated person. If he happened, through force of circumstances or through choice, to engage in some other branch or speciality of engineering, or if he desired to pursue further the specialty that he followed in his undergraduate years, he was left largely to his own devices and judgment for guidance in pursuing his further education. Few signposts existed to guide him; and there were no goals toward which he could strive in his search for knowledge as he strove for his bachelor's degree.

The prevalent specialization or study of the more advanced applications of basic fundamentals in engineering courses cannot have been due to any single cause or condition, but rather to a considerable number of them. Courses were planned and designed by engineering educators, who were, in most instances, themselves authorities in some specialty. It is a natural tendency for specialists to predominate on engineering faculties. Names of outstanding specialists give prestige and standing. Younger faculty members can best achieve eminence and likewise advancement through acquiring a reputation as an authority in some specialty.

A further cause of this prevalent specialization lies in the popular demand for courses in specialized subjects when a particular specialty has reached a stage which indicates that it might currently offer a considerable amount of employment. For example, interest in irrigation during the early years of the twentieth century developed widespread demand in western schools for courses in irrigation engineering. Rapid development of the petroleum industry during the decade from 1920 to 1930 caused many schools to offer courses in petroleum engineering or geology. At the present time (1946) the increasing development of the aeroplane is causing many schools to offer courses in aeronautical engineering, even though the best informed persons in the industry forecast a postwar reduction in production to from 10% to 20% of present volume. When one school offers courses in a popular specialty, others follow, so as not to be accused of falling behind in the march of progress. The usual result of this practice is that a surplus of men soon develops who were trained in these specialized fields, and many graduates who entered them following commencement are forced to seek other fields or specialities. The specialized courses continue, however, long after the demand for them has subsided, since there is a strong tendency to build curricula around the particular qualifications of individual faculty members.

In considering the progress of engineering education, as it is possible to encompass it within the limited time that the student spends in school, one might liken it to a basic industrial progress in which raw materials are subjected to certain operations and processes, from which an initial product results. This product undergoes subsequent processing of many types, each depending upon the ultimate product desired.

A steel plant, taking iron ore, coke, and limestone, produces first pig iron and then steel billets. From the latter a wide variety of products are made, differing greatly in character, quality, and composition. Seldom if ever does the steel plant attempt to process the billets beyond rolling them into shapes, or to produce the ultimate product, which may range from automobile bodies to

pipe wrenches and watch springs. It produces a basic product which, however, is of such a character, composition, and quality that a more finished product can be made, through further processing, by plants which specialize in further processing for the ultimate markets.

Managers of the steel plants watch the ultimate market closely, forecast demands, and keep abreast of these demands both in volume and character of product. The engineering school that produces graduates who have majored in such specialities as structural engineering, power plant engineering, or design of gas engines may be likened to a steel plant that sought to produce not only steel billets, but also such finished products as automobile bodies or watch springs.

Specialty graduates are sometimes sought by employers to maintain the lower grades of responsibilities on their engineer staffs. In some few instances they are in demand when employers hope to groom a future manager or president; and they are always in demand, of course, when the economic requirements for engineers are greater than the available supply. In such cases, it is but natural for engineering faculties to feel satisfied that the type and character of the courses they offer meet the requirements of the market for engineering services.

Faculties have little contact with the market for engineers with from five to ten years or more of experience. If their members are engaged in outside practice, it is usually as specialists or in a consulting capacity where they have little contact with the rank and file of engineers who constitute the bulk of the profession. Employers seeking to engage older engineers secure them through sources other than alumni rosters and collegiate personnel placement offices.

The Engineering Profession

The engineering profession consists of those persons, almost entirely men (few women are found in engineering), who practice engineering. In general, it may be considered as a heterogeneous rather than a homogeneous profession, with very nebulous boundaries. The field which the practice of engineering

TABLE 1.—Number of Technical Engineers Listed in

Line	Engineers	(a) Total Number Listed				(b) Number Per		
		1910	1920	1930	1940	1910	1920	1930
1	Civil (and surveyors)	52,033	64,660	102,086	99,437	56.6	61.2	83.2
2	Electrical	15,278	27,077	57,837	54,895	16.6	25.6	47.2
3	Mechanical (and others)	14,514	36,689	54,356	106,514	15.8	34.7	44.2
4	Mining	6,930	6,695	11,970	9,236	7.5	6.2	9.8
5	Totals (averages in parentheses)	88,755	135,121	226,249	270,082	96.5	127.7	184.4

covers is so broad and so diversified, the professional educational background and training of those individuals engaged in it covers such wide extremes, and its members so frequently shift their employment, character of activity, and functions—that very little homogeneity exists. Furthermore, practice of en-

gineering involves activity in so many other lines that it is extremely difficult in many instances to determine whether or not a particular activity falls within or without such practice.

General Traits and Characteristics of Engineers.—In spite of this general heterogeneity of the profession and its more or less nebulous boundaries, engineers as a group of individuals do have certain more or less common traits and characteristics which they have developed as a result of their education and the type and character of their professional experience. For the same causes, they are also lacking in certain attributes.

On the credit side it may be stated that engineers in general are resourceful, hardworking, ingenious, and persistent, and have a great devotion to their work. On the debit side they are extremely narrow in their interests and outlook, and highly individualistic. An engineer is the original "man from Missouri" who "has to be shown" everything.

Engineers cannot express themselves well—too few graduate engineers can write a good report or make out a salesmanlike application for a job, even after they have been out of college from ten to twenty years. Many a graduate, capable of bigger things, never has an opportunity to do them because of his inability to "sell" his capabilities to those who might desire to utilize them. Engineers lack a "dollar" sense, seldom appreciate the relation between the cost, value, or profit. Their habit of basing their work, opinions, and conclusions on factual data, although it is an excellent one, tends to cause them to be too strongly influenced by the details or tangible aspects of a problem and to ignore the intangible phases. They fail to see the broad general features. Engineering educators should recognize the existence of these items on the debit side more than they have done and make a stronger attempt to reduce this general debit balance.

Number of Engineers.—Since the very fundamentals of engineering rest on the systematic collection, analysis, and interpretation of factual data, the paucity of available data pertaining to engineers and to the engineering profession is remarkable. Even data concerning the number of persons practicing

THE FEDERAL CENSUSES OF 1910 TO 1940, INCLUSIVE

100,000 POPULATION		(c) PERCENTAGE OF TOTAL IN EACH BRANCH				(d) PERCENTAGE INCREASE OVER 1910				Line
1930	1940	1910	1920	1930	1940	1910	1920	1930	1940	
83.2	75.5	58.6	47.9	45.1	36.8	100.0	124.3	196.3	191.0	1
47.2	41.7	17.2	20.0	25.6	20.3	100.0	177.1	378.5	359.1	2
44.2	81.0	16.4	27.1	24.0	39.5	100.0	252.8	374.4	734.0	3
9.8	7.0	7.8	5.0	5.3	3.4	100.0	96.6	172.7	133.3	4
184.4	205.2	100.0	100.0	100.0	100.0	(100.0)	(152.2)	(255.5)	(304.2)	5

engineering are of poor quality. For several decades the Federal Census of Occupations has listed the number of "technical engineers" gainfully employed in the United States, and Table 1 shows these figures for the past four decennial censuses. Line 3, Table 1, listing mechanical engineers "and others" includes

engineers in all categories not otherwise mentioned in the table. The 1940 data include engineers seeking work but not those employed in emergency work. The total number of persons in each category listed (except mining engineers) has increased at a rate greater than the increase of national population, mechanical and other engineers showing the greatest rate of increase. Proportionately, the number of civil engineers and surveyors, listed together until 1940, decreased more than one third and that of mining engineers more than one half. Electrical engineers increased slightly, and mechanical and other engineers increased two and one-half times.

The 270,000 technical engineers listed in the 1940 Census cannot be considered as including all persons embraced in the practice of engineering in the United States. There are large numbers of persons listed under other occupational classifications who, without question, are following the practice of engineering. For example, the 1940 Census lists 9,236 mining engineers. Table 2

TABLE 2.—NUMBER OF PERSONS LISTED IN OCCUPATIONS AMONG WHICH MAY BE FOUND A CERTAIN PROPORTION OF TECHNICAL ENGINEERS (CENSUSES OF 1910 TO 1940, INCLUSIVE)

Item	Description	1910	1920	1930	1940
1	Chemists, assayers, and metallurgists...	16,273	32,941	47,068	59,333
2	Designers, draftsmen, and inventors....	47,449	70,651	102,730	107,286
3	Mine foremen and inspectors.....	23,338	36,931	34,286	33,457
4	Mine operators, officials, and managers..	34,325	25,234	30,896	31,672
5	College presidents and professors.....	15,668	33,407	61,905	65,627
6	Totals.....	137,053	199,164	276,885	297,375

shows that in 1940 there were 297,375 persons employed in occupations either closely, or generally, allied with mining engineering, or with other branches of engineering. Without question there were a considerable number of mining engineers, among these 297,375 persons, and also large numbers of engineers in other classifications. Similarly there are many other occupational groups more or less closely associated with technological activities, such as employees of public agencies, railroads, power companies, contractors, and industrial plants, among whom will be found many engineers. It is quite probable that the actual number of persons who could have been properly enumerated as practicing engineers in 1940 was several times the number listed as technical engineers in the census of that year.

This uncertainty as to the number of engineers in the United States is not in any way the responsibility of the Census Bureau. The enumerators of this bureau are not supposed to be skilled in personnel classification; they must accept the occupational classification given them, and in most instances they secure information as to the working members of a family from the housewife, whose knowledge of the occupation of her menfolk is usually ascertained from the title of the position which they hold, such as foreman, inspector, draftsman, designer, surveyor, or engineer.

An amusing incident illustrative of the difficulty of listing properly the occupational status in the census enumeration came to the writer's attention a few years ago. A friend of his, an engineer, was collecting occupational data in connection with a market survey in a large city, and was surprised to find some two hundred civil engineers listed as residing in a certain area of that city largely occupied by colored persons. Closer investigation disclosed the fact that a large number of colored drivers of garbage collection trucks and their assistants, who lived in the area, had given their occupations to the enumerator as sanitary engineers. They were employed by the Bureau of Sanitary Engineering of the City Engineering Department and, since sanitary engineering was a subclassification of civil engineering, they had been enumerated under the latter heading.

Composition of the Engineering Profession.—The diversity of activities, responsibilities, and functions in engineering practice is great. To describe, properly, the exact place that a person occupies in the practice of engineering, his duties must be classified under each of the following:

A. Horizontal Classification.—The position must be segregated as to the major branch of engineering under which it falls, and then under the particular specialty or subspecialty of the major branch.
B. Vertical Classification.—The relative importance, difficulty, and responsibility of the duties performed, which may range from subprofessional activity upward through the most complicated theory and practice, and the training and experience required to perform them must be determined.
C. Functional Classification.—The type and character of activity followed, such as design, construction, research, operation, sales, consultation, or administration, must be established.

Civil service and personnel administrators have made considerable progress in this type of "job specification," but the resulting number of individual classifications has reached vast proportions.

Seldom does an engineer remain in a given classification for any great length of time. The nature of his activity is one of constant change, such change frequently occurring in each of the three groups of classifications.

The 1940 Federal Census, for the first time, presented certain data relative to education, age, and income (from salaries and wages) for certain occupational groups, including technical engineers. Although there is no way of determining whether the data pertaining to this group are representative of those for the entire engineering profession (including engineers who were listed as following other occupations), as given, they do develop some interesting facts pertaining to a group who compose a substantial segment of the engineering profession. The census data are presented for that purpose, rather than as representative of the entire profession. Data for certain other occupational groups are also presented for purposes of comparison.

Education.—Table 3(*a*) contains data pertaining to educational qualifications of certain categories listed in the 1940 Census report under "Technical

TABLE 3.—PERCENTAGE ANALYSIS OF THE TOTAL NUMBER OF PERSONS IN THE 1940 CENSUS WHO WERE ENGAGED IN CERTAIN TECHNICAL AND OTHER OCCUPATIONS

Item	Description	Technical Engineers					Other Professional and Subprofessional					Total professional and subprofessional
		Civil engineers	Electrical engineers	Mechanical engineers	Other engineers[a]	All technical engineers	Chemists, assayers, and metallurgists	Draftsmen and designers	Architects	Lawyers and judges	Physicians and surgeons	
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
	(*a*) EDUCATIONAL QUALIFICATIONS											
	College Course Completed:											
1	Four years or more	58.6	60.7	59.6	71.1	61.3	58.9	15.5	56.0	80.2	93.5	55.8
2	Three years	16.6	11.3	12.4	11.7	13.5	13.8	18.7	16.5	10.0	3.1	14.8
	High School Course Completed:											
3	Four years	12.8	12.2	12.7	8.6	12.1	14.0	35.7	15.3	4.8	1.2	13.8
4	Three years	4.7	5.6	5.7	3.9	5.1	6.3	15.5	4.5	1.6	0.4	6.6
5	No years	7.0	9.4	9.2	4.7	7.6	6.6	14.0	7.3	2.8	1.3	3.6
6	Not reported	0.3	0.8	0.4	0.0	0.4	0.4	0.6	0.4	0.6	0.5	5.4
	(*b*) AGE GROUPS											
7	Median age, in years	41.9	40.3	41.3	37.2	40.8	33.6	34.4	42.0	44.1	43.1	38.7
	Percentage Less Than:											
8	Twenty years	0.0	0.0	0.0	0.1	0.1	0.6	1.9	0.2	0.0	0.0	0.6
9	Twenty-five years	3.4	4.5	5.2	9.4	5.1	13.9	16.2	3.6	1.6	1.0	8.0
10	Thirty-five years	27.6	30.0	30.0	43.8	31.1	56.1	52.1	28.6	31.1	28.1	40.4
11	Forty-five years	60.0	68.0	62.0	71.9	63.8	80.3	77.7	55.4	58.1	52.1	66.4
12	Fifty-five years	84.8	89.5	86.2	90.0	86.9	93.2	92.3	81.5	77.2	71.3	84.4
13	Sixty-five years	97.0	98.5	97.3	98.0	97.6	98.5	98.5	94.2	90.8	88.3	95.0
14	Seventy-five years	99.7	99.9	99.7	99.8	99.8	99.8	99.9	99.3	98.2	97.9	99.3
15	Older than seventy five	0.3	0.1	0.3	0.2	0.2	0.2	0.1	0.7	1.8	2.1	0.7
	(*c*) SALARY GROUPS[b]											
	Salary Range[b] (Dollars)											
16	Less than 100	6.7	4.5	7.0	8.8	6.6	5.9	5.4	37.6	56.9	65.1	...
17	100 to 600	1.9	2.2	2.3	3.4	2.3	5.0	5.8	2.6	1.4	4.1	...
18	600 to 1,200	4.9	4.5	4.8	5.8	4.9	10.8	14.6	5.1	2.5	2.6	...
19	1,200 to 1,600	7.6	7.8	6.8	7.2	7.3	15.6	17.3	5.1	2.7	1.5	...
20	1,600 to 2,000	12.2	11.2	9.8	11.9	11.2	15.3	16.8	5.3	2.1	1.1	...
21	2,000 to 2,500	21.3	16.8	17.4	18.1	18.6	15.8	19.3	9.8	3.7	1.9	...
22	2,500 to 3,000	13.1	12.1	11.8	10.0	12.1	8.4	9.4	7.9	2.5	1.3	...
23	3,000 to 5,000	24.0	28.5	26.1	21.3	25.3	14.9	8.2	14.9	8.2	5.6	...
24	More than 5,000	5.8	9.5	10.9	10.7	8.9	5.6	1.1	5.0	9.6	6.0	...
25	Not reported	2.5	2.9	3.1	2.8	2.8	2.7	2.1	6.7	10.4	10.8	...

[a] Including mining, industrial, and chemical engineers. [b] Percentage of the total number reported for the one year 1939 whose income from wages and salaries was within the salaries ranges indicated. This does not include income from professional fees and partnerships, or from other kinds of work or sources of income.

Engineers," and to certain other occupational classifications. Technical engineers are somewhat better educated than the average of the professional and semiprofessional group, but they fall behind the members of the medical and legal professions. Three out of four technical engineers have completed three

years of college, and three out of five have completed four years or more. As a member of the California State Board of Registration for Civil Engineers, the writer reviewed the professional biographies on about 6,000 applications for registration. Many of these applicants had received degrees from colleges everywhere in the United States and many of them were engaged in branches of engineering other than civil. From this experience, the writer is inclined to believe that the ratios in Table 3(*a*) would be somewhat reduced if applied to the entire number of engineers in the United States.

Age.—Table 3(*b*) indicates that technical engineers are somewhat older than the average of the professional and semiprofessional group, but slightly younger than the members of the medical and legal professions. A significant fact revealed in this table is that one third of the total number of technical engineers were within the range of from thirty-five to forty-five years old, whereas only one quarter of the members of the medical and legal professions were within this age bracket. Furthermore, although 86.9% of the technical engineers were younger than fifty-five years, only 77.2% of the lawyers and judges, and 71.3% of the physicians and surgeons were less than this age. This would indicate that a substantial number of technical engineers leave the strict practice of engineering during their forties, either to assume more or less non-technical administrative duties in engineering or industrial organizations, or to leave the field of engineering completely. A much higher proportion of the members of the medical and legal professions remain in their respective professions throughout their active life.

Income.—Table 3(*c*) gives the relative proportion of those employed as technical engineers and in certain other occupational classifications in various income brackets, such income being derived solely from wages and salaries. The census report states that very few of those listed as receiving incomes of less than $100 per year from these sources reported any income whatsoever from wages and salaries, indicating that they undoubtedly were engaged in private practice and received their income almost entirely from professional fees. This would indicate that about one out of fifteen technical engineers was engaged in private practice as against six out of sixteen architects, five out of eleven lawyers, and two out of three physicians and surgeons. It might reasonably be assumed that a substantial number of those listed as receiving an annual income of less than $1,600 from wages and salaries were engaged primarily in private practice.

Effect of Professional Registration.—According to the latest reports, there are about 70,000 registered professional engineers in the United States, registration statutes being effective in all but three states. This number is only one quarter of the total number of technical engineers listed in the 1940 Census, and probably less than from 10% to 15% of the actual total number of engineers in the United States. Professional registration has been effective and accepted by other professions such as law, medicine, and dentistry for many years, and has served to identify in the mind of the public at large what constitutes the practice of these professions. For this reason basic data pertaining to them are far more comprehensive, conclusive, and definite than are data pertaining to engineers and engineering. The attitude of the profession toward engineering

registration (or at least that of its leaders) was one of antagonism at first, but later developed into one of more or less indifference (or at least of only mild interest). This is more or less typical of professional attitudes toward other social and economic problems confronting the engineering profession.

The necessity of admitting to practice (when registration becomes effective) many persons of "borderline" qualifications, the wide variation in the standards maintained, and the wide variation in the diligence of enforcement of registration by the boards of the various states, combined with this professional indifference, has caused the results of registration to fall far short of those anticipated by its early proponents. It is a long-range program, however, and its full effectiveness will not be felt for several decades. In general, engineering educators have supported the movement, but not so actively that one of the strongest ambitions of every engineering graduate (recent or otherwise) is to receive, through it, official public certification of his technical qualifications as a member of his profession.

Organization and Structure of the Engineering Profession.—Definitions of engineering are legion, from the short ones found in dictionaries to the more complicated one encountered in engineering registration statutes. In essence,

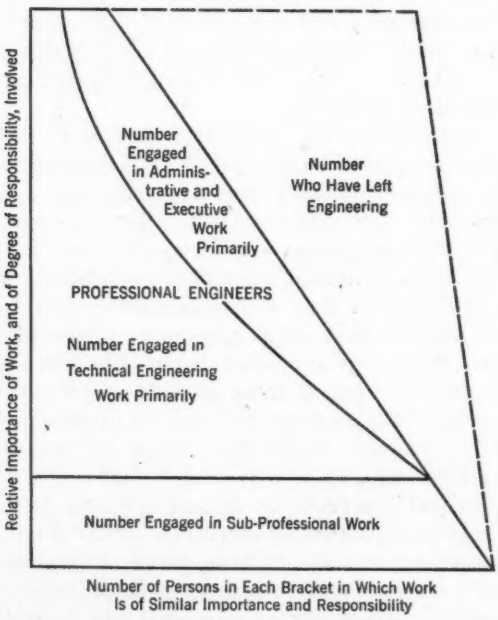
Relative Importance of Work, and of Degree of Responsibility, Involved
Number Engaged in Administrative and Executive Work Primarily
Number Who Have Left Engineering
PROFESSIONAL ENGINEERS
Number Engaged in Technical Engineering Work Primarily
Number Engaged in Sub-Professional Work
Number of Persons in Each Bracket in Which Work Is of Similar Importance and Responsibility

FIG. 1.—CROSS SECTION OF THE ENGINEERING PROFESSION (NOT TO SCALE)

however, engineering is applied science—the application of scientific knowledge, pertaining to the properties of matter and the sources of power, to the creation of structures, machines, and manufactured products. Half a century ago, these products could be created by a small organization, few members of which were

considered specialists. Today the field of engineering knowledge and likewise the products created have increased to such an extent in size, complexity, and cost that a large group or organization (including subprofessionals, professionals, specialists, and administrators) is required to perform such a function. The current practice of engineering, except by those who serve in advisory or consulting capacities, is a mass-production activity, in which each individual serves as a cog in a machine and performs his specific duties as a unit of a much larger group. It is far more similar to the practice of banking or insurance than it is to that of medicine or law.

TABLE 4.—SPECIALIZATION OF CORPORATE MEMBERS, AMERICAN SOCIETY OF CIVIL ENGINEERS (ASCE) AND AMERICAN INSTITUTE OF MINING AND METALLURGICAL ENGINEERS (AIME), EXPRESSED AS PERCENTAGES

Society	Engaged in Engineering			Not engaged in engineering
	Their own specialty	Another specialty	Total	
	(1)	(2)	(3)	(4)
ASCE...	52	5	57	43
AIME...	36	6	42	58

A cross section of the engineering profession, as shown in Fig. 1, is typical of those of other organization activities, being triangular in shape. At the bottom are large numbers of persons in positions that involve limited technical responsibility or knowledge, with few in the top brackets, most of the latter being engaged primarily in executive or administrative duties. It is more or less peculiar to the engineering profession that, as the importance and responsibility of the duties of the positions held by its members increase, the proportion of time spent in strictly technical activities decreases. This is not true of such professions as law, medicine, or dentistry.

Furthermore, an increasing number of engineers leave the profession as they advance in it. Table 4, based upon a 5% sample of the corporate membership of the American Society of Civil Engineers and the American Institute of Mining and Metallurgical Engineers, uniformly distributed throughout the 1937 membership roster of these societies, indicates this. When it is realized that all members listed as not following engineering had followed it for from at least six to eight years prior to their admission to the corporate grade in their society and had had at least one year in responsible charge of engineering work (which fact indicates that at the time they entered such grade they must have intended to follow engineering as a career), the results shown in Table 4 are rather significant.

THE PRACTICE OF ENGINEERING

Subject Matter.—Whereas a person engaged in the professions of law, medicine, banking, teaching, art, or drama deals primarily with the subject matter of his profession throughout his practice, a person engaged in the practice of engineering usually finds himself involved more and more with subjects in other fields as he advances in his profession. These subjects may be grouped under three headings, matter, men, and money—the three M's. Under the heading of "matter" are included technical subjects—problems of an engineering character. The heading, "men," would include subjects pertaining to human beings—activities, traits, habits, characteristics, and institutions. The

heading of "money" would embrace subjects in the fields of economics, finance, and business.

A young engineer, just graduated, finds himself dealing with more or less routine, or mechanical, matters; but, as he acquires knowledge, experience, and maturity, he is assigned problems involving professional knowledge and judgment. Working as he usually does, as a member of an organization, he continues to advance professionally; but he also begins to assume administrative duties, at the same time depending on his subordinates more and more for technical results and opinions. He is then dealing with both "matter" and "men." Unless he develops into a specialist, he also begins to encounter problems in the field of money if and as he continues to advance. He becomes responsible not only for the technical correctness of his work and that of his subordinates and for the efficiency of their output, but he becomes responsible for the cost and economy of the thing or things produced. If he still continues to advance, he enters the field of "management," the fourth M, which embraces the other three—matter, men, and money. In this field whether he operates in a large or in a small organization, he becomes involved in such problems as the relation between volume of production and absorptive capacity of markets, and as the cost of production in relation to profits, working capital, financing, taxes, interest, and dividends. He must retain his broad knowledge and judgment in technical matters, even though he forgets technical details; but he must also exercise knowledge and judgment in other fields.

Not all engineers progress through these three fields to the fourth one, many remain strictly in fields concerned with the first M—matter; but the highest rewards, except in the case of outstanding specialists, come to those who attain positions in the field of management. In addition to those who follow the foregoing course, and retain their connection and contact with technical activities, many leave the field of engineering entirely, and become engaged in nontechnical activities in which their engineering education and training in the approach to and the solution of problems, the orderly collection and assembly of factual data, its analysis, interpretation, presentation, and use prove to be of extreme value. Engineering educational institutions should recognize the foregoing condition in the subject matter included in their curricula.

Employment Conditions in the Practice of Engineering.—The engineering graduate, on entering his profession, is faced with the strong probability that his specialty, his employer, and his place of employment will undergo a considerable number of changes during his career, if he continues to follow his profession. Table 5, although based on data concerning members of the American Society of Civil Engineers, is probably representative of the situation in other branches of engineering. This table represents changes in employment in the case of one hundred Juniors and one hundred Associate Members of the Society between 1930 and 1940. Intermediate changes during this ten-year period were not noted. The names selected were of members who had entered their respective grade during the five-year period, from 1926 to 1930, inclusive, who were picked at random, but who were uniformly distributed through the membership rosters for the selected decade. Specialities and employers were based upon titles given in the roster.

It is significant that nearly one half of the Juniors and one third of the Associate Members either had left engineering during this period or had changed the specialities they followed in 1930; that three out of five Juniors and one half of the Associate Members had changed employers; and that nearly three out

TABLE 5.—CHANGE OF STATUS, AS ANALYZED BY THE RECORDS OF 100 JUNIORS (JUN.) AND 100 ASSOCIATE MEMBERS (ASSOC. M.), ASCE, BETWEEN 1930 AND 1940

Item	Description	Jun.	Assoc. M.
	(*a*) CHANGE IN SPECIALTY		
1	Abandoned engineering	11	5
	Remained in Engineering:		
2	Changed to another kind of engineering	36	29
3	Stayed in their original specialty	53	66
4	Total	100	100
	(*b*) CHANGE IN EMPLOYER		
5	No change in employer	40	49
6	Changed employer	60	51
	(*c*) CHANGE IN LOCATION OF EMPLOYMENT		
7	No change in location	43	58
8	Changed location	57	42
	(*d*) CHANGE IN SOCIETY STATUS		
	Advanced One Grade:		
9	Number advanced	100	36
10	Years required for advancement	7.0	6.8
	Advanced Two Grades:		
11	Number advanced	9	....
12	Years required for advancement	12.0	....

of five Juniors and two out of five Associate Members had likewise changed the location of their employment. Undoubtedly a higher proportion of members than that shown in Table 5 had left the practice of engineering during the decade, as those enumerated were those who retained their membership for the period, and members who had left the Society during the period because they had left the practice of engineering were not included. There appeared to be somewhat greater stability in employment conditions among Associate Members than among Juniors. Whether this lack of stability among Juniors was accelerated because of conditions created by the economic depression during the period cannot be determined; but, from personal observation, the writer is not inclined to believe that this was so to any great degree.

Rate of Advancement.—Another feature of interest revealed by Table 5 is the apparently slow rate of advancement by these members to more important positions. Slightly more than one third of those who were in the grade of Associate Member in 1930 had advanced to that of Member ten years later. The Member grade in the Society requires that an engineer shall have had five years of experience in responsible charge of important work—four years

more than that required for the Associate Member grade—and shall be qualified to conceive and design engineering work. It likewise requires at least twelve years in the practice of engineering. It does not involve any financial outlay for transfer in grade, nor any increase in annual dues. It has been the writer's observation that a high proportion of Associate Members of the Society are extremely ambitious to advance to the Member grade, and the small proportion of those who had made such advance would seem to indicate a lack of opportunity for achieving the qualifications necessary for such advancement. Of greatest interest, however, is the relatively slow rate of advancement of Juniors. Of the one hundred Juniors considered—

> Forty one required less than seven years,
> Twenty nine required between seven and ten years, and
> Thirty required more than ten years

—in which to advance to the grade of Associate Member. Theoretically, except for the age limit, a Junior of the Society, if he be a graduate of a recognized school, could attain the grade of Associate Member within a little more than two years after he is graduated. Few Juniors, if any, do so even upon reaching the minimum age (which is attained in from five to seven years after graduation). In the decade studied, advancement to the grade of Associate Member was optional until the Junior reached his thirty-third birthday, at which time he was dropped from membership if he had not advanced in grade.

During the three years from 1939 to 1941, inclusive, for every one hundred Juniors who did advance to the grade of Associate Member, there were forty three dropped for failing to advance on reaching the limiting age; seventy three were dropped for other reasons, and twenty one resigned—a total loss of one hundred and thirty seven. These figures indicate that the younger engineers, at least those who follow civil engineering and who have joined their own professional society, during this period at least, were making rather slow progress in their professions. This condition was recognized by the Society in 1942 when the limiting age for transfer from Junior to Associate Member was increased from thirty-three to thirty-five years.

In general, it is the writer's observation (based on a review of the professional biographies of from 6,000 to 7,000 engineers of all ages and lengths of experience who sought registration as civil engineers in California, or who sought admission to, or transfer in, membership grade in the Society) that the average engineer—at least if he follows civil engineering—does not attain a position in which he is called on to exercise independent engineering judgment or to assume engineering responsibility for anything except work of the simplest character, until at least from eight to ten years after he has graduated; and many do not attain such a position within that time. There are many exceptions to this statement, of course; but for all those who do make better progress there are a comparable number who require a longer period. During this period (which is at least from two to two and one-half times the period spent on the normal engineering course in college), a very substantial number of graduates will have changed specialties, employers, and location of employment, and a considerable number also will have left the practice of engineering to enter other fields. This period, during which the graduate has the oppor-

tunity to specialize in engineering, and likewise to acquire knowledge in other fields, should be recognized by engineering educators in designing their courses and curricula; and they should provide opportunity during this period for the guidance and assistance which graduates both desire and need.

Change in Character of Employment.—Another trend that became very significant during the decade from 1930 to 1940, at least in civil engineering, was the substantial increase in employment of civil engineers by public agencies. Table 6, based on a 5% sample uniformly distributed through the membership

TABLE 6.—TREND IN THE EMPLOYMENT OF ASCE MEMBERS

Item	Number, per 1,000 members, employed by:	1920[a]		1930[a]		1940[a]	
		No.	%	No.	%	No.	%
1	Private concerns	53.9	100.0	54.6	101.4	38.0	70.5
2	Public agencies[b]	23.9	100.0	22.6	94.5	45.8	191.8
3	Private practitioners[c]	22.2	100.0	22.8	100.7	16.2	72.9

[a] Under each year are listed the number (No.) in each 1,000 members who are employed as shown, and the percentage (%) of the 1920 value. [b] Excluding members in the armed forces. [c] Including employees of practitioners.

rosters of the Society for 1920, 1930, and 1940 indicates that, although there was little change in the relative proportion of Society membership employed either by private business or by public agencies, or engaged in private practice (or employed by engineers in private practice) between 1920 and 1930, in the following decade between 1930 and 1940 the number per 1,000 members employed by private concerns and engaged in private practice or employed by private practitioners, dropped almost one third. The number per 1,000 members employed by public agencies almost doubled. No doubt, some of this shift to employment by public agencies resulted from the economic depression of the 1930's but the outlook is for it to increase rather than to decline. The subject is worth further investigation.

The relatively slow advancement of Juniors may be due to the relatively large numbers employed by public agencies, mostly under civil service, where advancement is more dependent on seniority and increase in size of organization than it is in private employment.

THE MARKET FOR ENGINEERING SERVICES

The foregoing discussion and statistics have been presented to furnish a concise picture of the engineering profession and of the practice of engineering, for which engineering education is presumed to furnish the preliminary preparation. The statistical data are not complete; nor are they conclusive; but the size of samples is sufficient to make the results at least indicative of conditions and trends. The data also indicate the further research which might be made from existing information. If engineering education is considered as only the initial processing of a raw material to meet the subsequent needs of a potential market (and it is assumed that such initial processing must be followed by much further processing), two alternatives exist in the planning and design of engineering courses:

1. A rather complete knowledge of the character and volume of the present and the future market should exist, if the initial processing is to be refined in character, or

2. The processing should be of a character that will allow the product to adapt itself to the existing and future market, if information as to the character and volume of the latter is lacking or deficient.

If data were available as to the number of persons who had been employed in occupations involving a greater or lesser degree of engineering knowledge and experience over the past few decades, it might be possible to forecast, with reasonable accuracy, the character and type of existing and future markets for engineering services. Few data of this type are available, however, and little has been done with those that do exist. The result has been that large numbers of engineering graduates have been produced in the United States, each year for many years, by plants representing substantial investments and operating costs; and these plants have had little or no information as to the capacity of the market to absorb or to utilize their product in its latter stages of processing. They knew that their product was absorbed after initial processing, but had little knowledge of its fate, or of the requirements of the market for its services, subsequent to a period of from three to five years following such initial processing. Although quantitative data pertaining to the general volume, type, and character of market for engineering services are almost completely lacking, some information is available as to the character and type of product sought by the consumers who constitute this market—the employers of engineers.

The American Association of Engineers in 1940[2] published an analysis of "job orders" or descriptions and specifications given by prospective employers seeking men through the association's employment department in Chicago, Ill., during the previous ten-year period. Positions to be filled from these orders numbered into the thousands; and annual compensation ranged from a few thousand dollars upward to five figures. Most requests were from employers in the Great Lakes area, but a considerable number were from various parts of the United States, and the basic data available were of sufficient magnitude and distribution to warrant the conclusion that they represented a good cross section of employer requirements throughout the nation. In addition to requiring that the employer specify the technical training and experience, salary offered, and duties involved in the position to be filled, he was requested to specify other characteristics and traits desired in the prospective employee. In most instances employers gave as much weight to nontechnical as to technical qualifications.

A summary of the required qualifications showed the following to be prominently stressed:

Item	Qualification
a	A soundness in engineering fundamentals that would enable the employee to translate experience gained in one enterprise to a different type of industry, but in the same general branch of engineering;

[2] *Professional Engineer*, November, 1940.

b A habit of continuing study, not only of technical theory, discoveries, products, and processes as they are developed, but also of commercial developments;

c A cost consciousness that would allow the engineer to appreciate the cost of engineering and its results, in relation to the profit possibilities of the entire enterprise, and to understand the problems of the accounting department;

d An ability to handle men, which involves the abilities to secure the cooperation, confidence, and respect of subordinates, associates, and superiors; to inculcate in others the desire to do things for one and to follow one's leadership; and to lay out and plan future operations in such a manner that these operations, when undertaken, would achieve a desired result;

e An initiative and resourcefulness indicated by a desire to step out and solve problems without having to be urged by others to do so; and

f An ability to express ideas in written and spoken form, in a manner that would convey them not alone to other engineers but also to laymen.

The demand for technical graduates was not insistent, although there was an increasing desire throughout the period for some technical education. Where graduates were specified, stress was laid on good, rather than on high, scholastic records; and equal or greater emphasis was placed upon participation in extracurricular activities while in school—the latter being considered as sound training for leadership.

Although specialization in college courses was requested by some employers—where recent graduates were sought—most prospective employers stressed training in fundamentals, and broad application in the various fields of engineering, preferring to train their own employees in their respective specialities. Considerable opposition was voiced to scholastic specialization, on the basis that it tended to develop an inflexibility in the employee which did not readily allow his being transferred from one department to another in the course of his advancement. Some of the foregoing qualities sought by employers may be considered as being more or less a matter of inherent native traits or qualities; but all of them can be improved, and many of them developed by proper educational processes and instruction, which frequently will bring out potential qualities that otherwise would remain dormant.

Postwar Engineering Education

Advances in science and technology as a result of the war effort indicate a postwar market for engineering services greater in volume than that existing before World War II, with a demand for services of a more diversified, and in many instances of a more specialized, character. Other things can likewise be expected of this market—among them: (1) A longer period of study and training for engineers (because of the more complicated subjects involved in engineering practice); (2) the broader knowledge of a wide range of both technical and nontechnical subjects necessary to provide maturity of judgment; and (3) the greater demand in the increasingly industrialized postwar economy for executives and administrators who have had background and experience in engineering.

To meet the requirements of such postwar market, and to equip engineering graduates properly to cope with the situation adequately and to make the most effective contribution to the postwar economy, it would appear that the following changes in engineering education, as compared with those generally offered during the fifteen to twenty years prior to World War II, are indicated:

(*a*) Engineering courses should provide a broader scope of basic instruction in subjects that cover the entire field of engineering, should include in the curricula a greater proportion of nontechnical subjects of a character that will be of value to the engineer who may engage in executive or administrative activities, and should leave specialization primarily to the period following graduation.

(*b*) Provision should be made for the instruction—through night classes or correspondence courses, or in some other manner—of those students who desire to continue their studies in specialties or nontechnical subjects following graduation and who do not, or cannot, return for resident postgraduate study.

(*c*) Extensive research concerning the engineering profession, the practice of engineering, and the market for engineering services should be sponsored not only by engineering schools but also by engineering societies.

The first change suggested will unquestionably entail a drastic revision of curricula. Many courses in specialized subjects, given in the last two academic years, will have to be resident graduate courses, night courses, or correspondence courses. Many courses which are essentially training in skills will have to be discontinued, and likewise many nontechnical courses will have to be added. To the writer, it has always seemed rather fruitless for a student to take a highly specialized course in theory and practice in his senior year, and then wait for six, eight, or ten years following graduation before he has an opportunity to apply the theory or practice learned in such course in any capacity except that of a subordinate under immediate supervision. The excuse usually given for such courses is that, even if they are never utilized in later practice, they were good mind training. Granting that this be true, comparable training could be achieved through more courses in mathematics or science, the tools that the student will use in later life to aid him in increasing his technical knowledge. The specialized knowledge acquired in such courses, if needed, could be acquired during the six, eight, or ten years following graduation.

With the considerable changing from specialty to specialty that exists in engineering practice, it would seem that the student would be better equipped to make such changes if he had a broader knowledge of general engineering. He would not then have to go back so far to start acquiring basic knowledge of a new specialty when such a change was made.

Particularly in the first two years, many of the courses that seek primarily to impart certain skills, could well be eliminated; or the time spent upon them could be reduced, although the purpose for which they are offered could still be achieved. Such courses include surveying (essentially applied trigonometry and use of precise instruments), drafting, and certain shop courses. Ordinarily these are not of sufficient length to produce skilled craftsmen. They

could well be supplanted at least in part by courses which would help to prepare the student to express himself clearly and well, which would teach him something of human nature, of institutions, of the laws of economics, of money, and of business practice; and how to figure costs and to determine values. If the student is to follow a career that will cause him to deal with "men" and "money," as well as with "matter" (and most engineers do follow such a career at least part way) he should be given some basic instruction in these subjects while in school and should not have to acquire a knowledge of their first principles following graduation.

The student should also receive a realistic picture of the profession which he has chosen for a life career—not the glorified aspect usually presented by older alumni or distinguished engineers who address upper classmen at meetings. He is entitled to know what he is "getting into"—something of the organization of his profession, of its ethics, and of its code of practice; of the kind of life he might expect to lead five, ten, twenty, or more years after graduation if he follows this or that branch, specialty, or type of employment; of the probable rate of increase in compensation to which he may look forward in later years; and of the probable opportunities in various fields of activity.

He should be furnished information as to opportunities in business and industry for a person with a technical education and some engineering experience. Too many graduates, after a few years of experience in engineering, find that they do not like it, but feel that, since their training and education have been along technical lines, there is no opportunity for them in other fields. They then remain as misfits in this profession when they might have made a success in some other line.

An engineering course that would prepare its graduates to enter the bottom strata in any branch of engineering, that would give them a foundation in those subjects which are fundamental to business and industry, and that would emphasize training in the engineering or scientific approach to the solution of problems will produce graduates who meet the requirements of the postwar market for engineering services far more adequately than a course which attempts to produce a graduate who has been prepared to enter one branch or specialty in engineering.

Every engineering graduate realizes—either on graduation or very soon thereafter—that he has just begun to accumulate the knowledge which he will need before he attains a position of responsibility in his profession. When he was graduated, he had the habit of study and a keen desire to continue to increase his knowledge. Too often this was soon lost for the lack of a properly supervised and directed course of study or reading that he could, and would, follow if he had proper guidance plus a goal toward which he could strive. Even if he desired to increase his knowledge of the work in which he became engaged on graduation, he was met with such extensive literature that he became confused, did not know where to start his further study, and too often contented himself with merely reading current articles in such technical journals as came to his attention.

Many engineering schools offer extension courses; but either these seldom lead to a specific objective or they are designed to provide complete instruction over a period of years in a given specialty. In addition, these courses do not

set up any goal of accomplishment toward which the student can strive as he could for his engineering degree. The habit of study and the desire to increase knowledge is a very real asset in a graduate; it should be conserved and not allowed to dissipate, as too often happens. Only a small proportion of graduates can, or will, return for resident postgraduate study. Many of them are not able to attend night courses. They can enrol in correspondence courses, however, or pursue systematic reading courses which have been outlined under proper guidance.

If graduate instruction were planned and arranged so that the student, after completing a given course of study, taking an examination similar to those given by the better engineering registration boards, and submitting a thesis indicating his ability to perform original and responsible engineering work, was awarded a professional degree, many graduates of four-year courses would avail themselves of such opportunity, and the profession would benefit. Such courses would furnish preparation in specialized subjects offered in many undergraduate courses.

The foregoing suggestions would cause some changes in existing practices and habits of engineering faculty members; but they would probably result in an increase, rather than in a decrease, in the size of engineering faculties. Broad general courses covering the entire field of engineering would tend to increase the number of students returning for graduate work. Specialists who formerly gave instruction orally in their specialities would give more of it by correspondence and might have more time for research or possibly for outside practice. Because of the longer time over which they had contact with their students, the faculty would secure a better knowledge of the market for engineering services; and they would be able to exercise better control over courses and be better equipped to modify instruction to keep abreast of current needs.

Research concerning the engineering profession, the practice of engineering, and the market for engineering services—in brief, along the economic and sociological aspects of engineering—is sorely needed. As stated heretofore, this research is a responsibility not only of engineering educational institutions but likewise of engineering societies. Tools for this research have been developed during the past decade or two which allow a great amount of research to be done economically. The technique of scientific sampling has made great advances since 1935 enabling many basic data of a fully adequate quality for the purposes needed to be secured. Use of punch cards and tabulating machines has greatly reduced the cost of analyzing data.

Professional biographies of registered engineers and members of engineering societies, and records filed with civil service boards and commissions and with concerns employing large engineering staffs provide an untouched mine of information concerning the profession. New data are included in the 1940 Federal Census. The lack of such data in the past was probably due to the fact that engineers, engineering societies, and engineering instructors were far more interested in the technical aspects of engineering than in the economic or sociological aspects. Developments during the past few years indicate that engineering societies and schools must give thought and study to engineers as well as to engineering. Not only should the profession know something about itself and its members; but those who are preparing to enter it are entitled to this knowledge, and an obligation rests upon engineering schools and engineering societies to furnish them with it.

AMERICAN SOCIETY OF CIVIL ENGINEERS

Founded November 5, 1852

REPORTS

TOPOGRAPHIC SURVEYS

PROGRESS REPORT OF THE COMMITTEE OF THE SURVEYING AND MAPPING DIVISION ON TOPOGRAPHIC SURVEYS

SYNOPSIS

Information supplemental to that contained in the many excellent textbooks that have been written on surveying is presented in this report. It avoids, as far as possible, the repetition of textbook data which are concerned with the mechanics of survey instruments and survey operations.

The assumption is made that complete and trustworthy topographic information is vital to good engineering design when the use of land is involved. Since land use projects vary widely in their character and in the physical characteristics of the terrain, so also will there be a wide variation in the type of topographic information and in the manner of its presentation if the map is to render maximum service.

Effort is made to analyze the principal factors that influence the plan for a proposed topographic survey and the maps that will serve the requirements of a particular project. The following factors are analyzed: The interrelationship between map scale and map accuracy, the selection of the map scale and the contour interval, the function of control and its distribution, the different methods of developing topographic maps, the map coordinate system, and map correlation.

Specifications are submitted which will permit the production of maps of stipulated accuracies and types. A method of map classification based on map accuracy and a method of map nomenclature to be applied to maps of differing types based on their purpose or end use are also suggested.

PURPOSE AND SCOPE

This report is presented in the desire that it may be helpful to topographic engineers in their effort to render the maximum of service to their clients,

NOTE.—Written comments are invited for immediate publication; to insure publication the last discussion should be submitted by September 1, 1946. Progress Reports are published in *Proceedings* only

and to assist engineers engaged in design and construction to judge the adequacy of the map to serve as a basis for their work.

The committee hopes that the report will tend to raise the standards of topographic surveying and will help engineers to appreciate the fact that topographic mapping is a type of service that requires special training and that offers opportunity for the display of skill and imagination in the improvement of the quality of the service. Topographic mapping methods are not static.

Attention is focused herein on some aspects of topographic surveying which are often overlooked by many persons undertaking such work. The topics discussed are general, such as classification of maps, the selection of map scales and contour intervals, methods of developing topographic maps (with characteristics of each), the planning of the survey, and writing specifications.

In appreciation of the fact that many excellent textbooks and federal government bulletins treat, in detail, the description, care, and manipulation of survey instruments and their accuracy, this report does not attempt to invade this field, but refers the reader to his favorite textbook.

Function of a Topographic Map

The function of a topographic map is that of a tool fashioned by the topographic engineer to serve: The planning engineer or architect as a basis for the project design; and the construction engineer by facilitating his work of translating the planners' paper plan to the ground, for construction purposes.

If the topographic engineer is to fashion a tool that will serve these men adequately, he must have sufficient knowledge of both design and construction to plan and produce the map that will exhibit all pertinent data.

Classification of Topographic Maps

Topographic maps may be classified under three general headings or types dependent upon their use and map scales:

Small-Scale Information Maps.—Large areas are mapped at map scales varying from 1 in. equals 2,000 ft to 1 in. equals 1 mile, approximately, as illustrated by the map sheets of the U. S. Geological Survey, the Tennessee Valley Authority, and the U. S. Corps of Engineers. The procedure for preparing for these maps is well standardized (refer to the topographic instructions issued by these agencies).

General Topographic Maps (*Reconnaissance*).—For generalized studies, decisions as to suitability of areas, preliminary location of highways and utilities, approximate estimates of costs, and the determination of those areas to be included in detailed topographic maps, map scales will vary from 1 in. equals 400 ft to 1 in. equals 1,000 ft.

Detailed Precision Topographic Surveys.—For detailed studies and plans and for estimates of cost, map scales vary from 1 in. equals 20 ft to 1 in. equals 200 ft.

Selection of Areas to Be Covered by Different Map Types.—The entire area available for the project should be included in the general topographic map except those parts which inspection indicates to be unsuited for any present project use. Only those parts of the area, general studies of which indicate a

probability of use and for which use detailed plans will have to be made, are recorded in detailed topographic maps. Such detailed maps will cost from two to five times as much per unit area as will the general topographic map.

Selection of Map Scales

The character of proposed developments, for which topographic maps may be made, has as many variations as there are activities in the field of human endeavor. The importance of the selection of the proper map scale cannot be overstated. The kind of project and the kind of use for the survey and map information will determine, to a great extent, the scales to which each map should be drawn. It is essential that careful consideration be given to this part of the problem in order that economy in time and in the cost of production may be effected. When analyzing this problem, the engineers must always keep in mind questions of clarity, accuracy, cost, facility of correlation, and the physical factors that affect the choice of a map scale.

Clarity.—Facility with which the data can be interpreted is a ruling consideration for clarity. Obviously, map value is lost if it is so cluttered with symbols that the important data do not "stand out" clearly. The user "Can't see the woods for the trees."

Accuracy.—The interrelationship between map scale and map accuracy must be kept in mind constantly. Obviously, it is absurd to demand, in the location of a mapped point, that its position is to be correct to 0.1 ft when, because of the map scale, 2 ft is the minimum distance the eye can plot. When the scale is fixed, the limit of accuracy for the map is also fixed. Conversely, if the tolerance of accumulated error for a map is fixed, a limit is placed on the map scale, at the same time.

Cost.—The cost of a topographic survey increases rapidly with increase of map scale, because of increased amount of detail that can be shown with clarity and because of the increase in the accuracy requirement for each of the survey operations that comprise the finished product.

Facility of Correlation.—To facilitate the correlation of data contained in the different project maps, the topographic map, as well as all other project maps, should be either on the same scale or, if this is not feasible, on scales which are even multiples.

Physical Factors That Affect the Choice of a Map Scale.—Three factors influence the selection of a map scale:

(1) The number and character of cultural features that are to be shown and the frequency of their occurrence;
(2) The nature of the terrain, steepness of slope, and the intricacy of the topographic detail; and
(3) The contour interval necessary to depict the topographic detail adequately.

Field inspection and study of these factors will pay dividends.

Classification of Maps According to Accuracy

To be considered acceptable from an accuracy standpoint, maps should fulfil the conditions stated in the Appendix. Maps complying with standards

stated under Section C should be classed as of "first"-order accuracy and so designated by a marginal note on the map.

The purpose of many "reconnaissance" topographic surveys is such that first-order accuracy is not required or desirable. Since, often, for general studies, errors in position of 10 ft or 15 ft are in no way harmful and since in the same way errors of 10 ft in elevation do not affect the general plan, the specifications should be modified correspondingly. Maps of this type should be designated as of second-order accuracy and should carry a marginal note specifying the designation.

Spot Elevations

Contours often cannot record the complete "story." Elevations of summits, tops of banks, bottoms of depressions, bridges, and road and railroad intersections should be given in numerals. These are known as "spot elevations."

Maps Requiring Two Contour Intervals

Where the area to be mapped includes areas having two very different types of slope, it is often desirable to use a different contour interval on the flatter areas. To use the same contour symbol for contours having different intervals destroys a correct visual interpretation of the change in slope. It is recommended in such instances that the one adopted contour interval for the map be used over the entire map and that, on the flat areas, in addition to the regular contour, a dotted contour line representing the one-half contour interval be located and drawn on the map.

Map Tests and Written Specifications

The planning department of an engineering organization always demands speed—and more speed—from the survey department. No matter how great is this demand for speed, the engineer should never be tempted to start field parties before writing a complete set of specifications for each of the survey operations that enter into the construction of the completed map. Field tests should be run on each map sheet immediately after field mapping is completed. Both these precautions will pay dividends in economy of time and cost before the job is completed. It is to be remembered that the completion date is the important date—not the commencement date. The map must be a homogeneous product, trustworthy in all its parts. This cannot be accomplished if each party chief is left to his concept of his part of the work. Each party chief must have definite written specifications covering his part of the field work since each one will interpret the written word differently. Therefore, it is necessary that there be close supervision of the field work by the topographic engineer in charge. Map tests will show that some men are too meticulous and waste too much time in accomplishing a higher degree of accuracy than that necessary, whereas others will be too careless and so vitiate the final product. Field inspection and field tests are necessary to reduce these considerations to a common denominator.

Method of Developing Topographic Maps

The intent herein is to present a brief outline of methods in common use and to present their characteristics. For details concerning instruments and methods of operation, the reader is referred to textbooks on surveying and on photogrammetry.

Systems of horizontal and vertical control established by ground surveys form the foundations or skeletons for the construction of topographic maps, regardless of the method of filling in the topographic information that transforms these skeletons into vital maps. Filling in topographic detail may be accomplished by:

(*a*) Plane-table method (entirely ground method);
(*b*) A combination use of plane-table and vertical aerial photographs;
(*c*) Stereophotogrammetric methods based on the use of overlapping vertical photographs and stereoscopic plotting instruments of various types; and
(*d*) Transit-stadia method (entirely ground method).

Study should be made of the specific conditions prevailing on each job before a decision is made as to the method that is to be used. Some one method is better suited to a particular set of conditions. No one method is a topographic cure-all, surpassing all others under all conditions.

(*a*) *The Plane-Table Method.*—In the plane-table method, the control data are platted on map sheets (usually 24 in. by 31 in.) the size of the plane-table boards. These are then sent to the field where the topographic information is obtained and platted on them. Direction is determined by pointing the telescopic alidade at the object or the rod. This direction is translated to the paper by drawing a penciled line along the edge of a fixed ruler attached at the base of the alidade. Distance is obtained either by stadia or by chaining, depending on the requirement of the map scale. Elevations are determined either by direct or by trigometric leveling, using the alidade for this purpose. The located points, with their elevations, are platted on the map immediately and the contours and other topographic features are drawn in while the topographer looks at the features to be expressed. The principal characteristics of the method are:

(1) Fidelity of topographic expression;
(2) Economy of the number of located points necessary to maintain a stated standard of accuracy;
(3) Elimination of office computation, interpretation, and platting of copious sets of notes sent by the field men to the office and the elimination of opportunity for error in the plotting of data from the notes made by another person;
(4) Since the map is drawn coincident with the location of the data, visual inspection of the map and of the ground showing the topographer just where the next rod is needed to develop the map properly, thus insuring lack of duplication and lack of necessary rodded points;

(5) Immediate detection of errors that may have been made and immediate opportunity to search for, and correct, the error; and
(6) Successful use at any map scale for areas of any magnitude or on any character of terrain.

There are conditions under which this method will not be as economical in time or cost as would some other method.

Commenting on characteristic (5): Plane-table traverses for the location of data are run in closed circuits, originating from, and closing upon, control points already platted on the sheet. The error of closure of the platted plane-table traverse indicates whether the error is greater than the tolerance stated in the specification.

(*b*) *Combination Use of Plane-Table and Aerial Photographs.*—In this method overlapping vertical photographs are obtained. The position of the ground control points are identified on the photographic prints. By comparison with the plotted position of these control points, the photographs are brought to correct map scale. The photographic data are then transferred, photographically, and in nonphotographic blue, to the plane-table sheets. Correct position of the transferred information is attained through the use of the horizontal control which has been previously platted on plane-table sheets. The plane-table map sheets must be mounted on aluminum or zinc, to prevent undue paper distortion during the photographic process of transferring the information from the aerial photographs to the plane-table sheets. The transfer process is a wet process. The plane-table sheets are then sent to the field for the location of the contours by the plane-table process. The topographer at the same time proofreads the sheets for any misinterpretation of the planimetry which was transferred from the photographs. He pencils on the blue line transfer, all data which are to appear on the finished map, such as roads and buildings. These map sheets are then returned to the office and the penciled data are inked in black. Since all other data on the sheet are in nonphotographic blue, they disappear in the reproduction process. The principal advantages of the method are:

(1) Fidelity of detail of planimetry and contour expression;
(2) Reduction of the number of rod readings necessary; and
(3) Requirement of no expensive equipment, beyond that usually in the possession of topographic engineers.

Since all planimetry is already located on the sheet, the only rod readings necessary (advantage (2)) are those to determine critical elevations necessary to contour location. As compared to the plane-table method, the rod-reading time in the field will be reduced by 60%, while maintaining the same map accuracy.

A disadvantage is that the method is limited to areas where the topographic relief of the area covered by a photograph does not exceed 150 ft if the map scale is "1 in. equals 800 ft." The method is well suited to the mapping of large areas having gentle relief, such as river valleys, flood control, and irrigation projects.

(c) *Stereophotogrammetric Mapping Methods.*—The method of filling in topographic detail by the use of stereoscopic plotting instruments requires the possession of costly equipment and of personnel trained in the use of such equipment. It should be attempted only by engineers thoroughly trained in photogrammetry. A few colleges teach photogrammetry and there are three or four excellent textbooks on the subject. Most of the topographic mapping by this method has been done by federal mapping agencies and by two private concerns who have the equipment and trained personnel. If private engineers desire to use the method it would be best to enter into a contract with one of these companies. Definite written specifications covering the map scale and the contour interval and a statement of the required map accuracy should be incorporated into such a contract. Enumeration of the features to be shown in the map should include the coordinate grid lines and the control stations with their designations, bench marks, and spot elevations. The nature of the tests that will be applied should be stated to insure that the map delivered is in compliance with the specifications in the contract. The method is capable of an accuracy suitable for detailed precision surveys only in those areas where the ground surface is visible from the air. Where the ground is obscured by dense brush or timber, or where the surface, although visible is flat and monotonous in appearance with little or no visible relief, the contour location will be difficult and in most cases faulty.

(d) *Transit-Stadia Method.*—As indicated in the name, the topographic detail in this method is located by transit-stadia measurements. The information is recorded in field notebooks, the data are computed and platted in the office, and the contours are drawn by interpolation between platted elevation points. In nature, slopes are irregular; they are uniform only for short distances. This method requires many rodded elevation points and very explicit notes if the interpolated contours are to express the topographic detail faithfully.

Horizontal Control Factors Determining the Use of Different Classes

Horizontal control often has two purposes—to control the accuracy of the topographic survey and to serve as a basis from which to set the stakes for construction. The methods of establishing control are "triangulation" and "transit traverse." Transit traverse is classified according to accuracy into first order, second order, and third order. Decision as to the amount of triangulation and the order of traverse that should be specified for a particular survey depends on: The map scale and the stipulated map accuracy; the extent of the area to be mapped; and the character of the terrain. Decision as to the accuracy of the control cannot be made until the map scale and the map accuracy have been decided. If the tolerance of error in platting is set at 1/50 in. on the map scale and if the map is to be at a scale of 1 ft equals 200 ft, then the permissible accumulated error of traverse would be limited to 4 ft. Should inspection of the area show that the principal traverse circuit will have a length, between existing control monuments, of 15 miles (78,900 ft), the limiting error of 4 ft requires a traverse with a closing error not to exceed 1:19,700 and a first-order traverse should be specified. However, if the terrain is such that

triangulation is practicable and if inspection shows that the probable length of traverse between triangulation stations will not exceed 3½ miles (18,480 ft), a limiting error of 4 ft requires a traverse, the closing error of which does not exceed 1:4,620 and a third-order traverse may be specified.

If the length of the secondary circuits (run between control points of the primary circuit) can be kept to less than three miles, they may be of the third-order accuracy; but, if the control is also to serve for construction staking, the traverse must be second order or better.

Permanency of Survey

No surveyor can afford to have his reputation deteriorate with the decay of the wood "hubs" or stakes that he has left behind him. He should give permanence to his work through proper monumentation. Not to do so is as unfair to his client as to himself. It is also unfair to the generation of surveyors that are to follow him.

Map Reproduction

Much of the accuracy of a map may be lost through its manner of reproduction. Why does any engineer expend the time and money to make a map whose limit of accumulated error throughout is 1/50 in. on the map scale and then lose this accuracy many times over through reproducing it by tracing linen and blueprinting, in which the over-all error may be as much as ¼ in. Tracing linen alone may change scale between morning and night by more than 0.10 in.

A film positive, made on low shrinkage film or on glass, at map scale, may be made by photography from any properly penciled map sheet. Prints from these film positives made by the "ozalid" or other dry reproduction processes will be much nearer to exact scale than blueprints from tracings on linen. The cost will be less than preparing the linen tracing. Maps of large and important projects (especially where many copies are needed) should be reproduced by photolithography or by engraving.

Map Symbols

Standard map symbols have been adopted by the federal bureaus. These symbols should be used by all topographic engineers.

Map Correlation

Facility of correlation of all map data for an area is important to every engineer and to those of future generations. The relationships in position between projects separated by distances (often considered too great to be of moment) are becoming more and more important. The state system of plane coordinates is being legalized by many states. If private engineers will refer position on their maps to this system, the correlation of all maps can be readily effected and much duplication of survey work be prevented.

Recapitulation of General Analysis

Each topographic survey will present different problems. Each one demands an analysis of the existing conditions that affect survey procedure. This report segregates five premises for discussion:

1. Use determines the permissible error in a map and affects the map scale and contour interval.

2. Having determined the required map accuracy, specifications should be drawn which will permit a map of that accuracy to be produced. Good engineering does not permit the map to be of either higher or lower accuracy.

3. Definite specifications for each component survey operation must be written and a detailed plan must be prepared covering the sequence, the timing, and the number and type of men needed for each survey operation to complete the job at a fixed date. The survey should be so planned that it will be a continuing manufacturing process, each component part of which operates concurrently, producing a steady flow of finished product.

4. Measures must be utilized to give the survey permanence on the ground and in the office and to make easy the correlation of all project maps, with each other and with those of distant areas.

5. Numerous methods of making topographic surveys are in common use. Each method has advantages under certain conditions. No one of these methods is best suited to all conditions.

Respectfully submitted,

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January 16, 1946

APPENDIX

RECOMMENDED STANDARD SPECIFICATIONS FOR TOPOGRAPHIC SURVEYS AND MAPS

SECTION A. GENERAL SPECIFICATIONS APPLYING TO ALL SURVEYS AND MAPS

A-1. Datum.—On all surveys, where possible, horizontal position is to originate from, and be closed upon, existing government control. Vertical position is to be referred to sea level datum, as established by government bench marks. When government or state control monuments are within five miles of the project, the surveys should be tied to this control.

A-2. Coordinate System.—Horizontal position is to be referred to a rectangular system of coordinates, the State Plane Coordinate System, where possible. If this is not possible, then an assumed value for the zero of coordinates is to be used. The azimuth of the axes is to be referred to the true meridian, determined either by observation on Polaris or by the sun as directed by government bulletins or modern textbooks on surveying.

A-3. Map Drafting.—Maps are to be drawn on standard sheets, the actual size and format of which shall be predetermined. The map is to show a recti-

linear projection, the grid lines of which are to be spaced 5 in. apart. The edges of the map shall be parallel to the grid system. Map titles are to be placed in the lower right-hand sheet corner between the neat line and the cutting line. Map titles are to show plainly whether position is referred to the State Plane Coordinate System and to sea level datum or whether these are assumed. If position is referred to the state system, the title must designate the name of the control point to which the survey is tied and the coordinate values and the elevation. This is important in case there should be later adjustment of these values by government survey bureaus. All maps must carry the date on which made, the name of the person who made them, and the name of the person or persons responsible for accuracy.

A-4. Station Numbering.—Each control station in the field shall be numbered on the field guard stake when it is set and this number, shown in the map, shall designate the station. The numbers are to be continuous in the order in which the station has been established. There is to be no duplication of station numbering on a project. Should an error be made in the numbering which gives the same number to two stations, the second station so numbered may be designated by adding to the number a letter of the alphabet. There is to be no changing of station numbering after the station has been observed, by either transit party or level party.

A-5. Field Survey Methods; Control System.—Angular closure errors of triangulation and traversing, and closure errors of level circuits, shall be checked in the field and shall be shown clearly in the notebooks. Traverses and levels shall be run in closed circuits. All angles observed on traverses must be doubled and the mean of the double value is to be used. All chained distances are to be checked by an independent method of measure (such as stadia) to guard against any miscount in chaining. These readings are to be recorded. Where discrepancies (indicating chaining error) between the chained distance and the stadia reading occur, the distance must be rechained at once. The correction made as a result of rechaining is to be placed under the original distance, and a line drawn through the erroneous one. There is to be no erasing of the original notes either of chaining or of the stadia reading.

A-6. Notebooks.—Notebooks are to be filed in the office of the Engineer of Surveys. Field notes must show, on each page, the name of the party chief and personnel of the field party, the type and location of the work, the weather, and the date. The book is to be clearly indexed. Each book is to be labeled, indicating the type of work, and is to be numbered consecutively for each class of work. Instrumentmen must take particular care that the station designation in their notebook is the same as that shown on the station "guard stake" on the ground. Computation books must designate clearly the character of each computation, the date, by whom made, by whom checked, and the page and number of the field book from which the original data were obtained. This information should be shown on each page. The book must be fully indexed. A bench-mark book must be kept showing the adjusted elevations of all bench marks, with a description of each bench and showing the page and number of the level book in which the elevation was determined.

A-7. Computations.—All computations are to be checked by an independent computation by a different individual before results are accepted.

A-8. Monumentation.—Control stations are to be monumented. Monuments must be carefully set, well tamped, and allowed to settle for at least three or four days after setting before either traverse or levels are run over them. It is preferable that they not be run over until there has been a rain between the time of setting and the time of locating. Where speed is such a factor and rain may not be waited for, then the earth around the monument should be well watered and tamped at the time of setting the monument. Monuments should at least be long enough to extend below the frost line, and should be 5 in. or more in diameter at the top, with a brass plug 1 in. in diameter and 3 in. long centered in the monument top. The plug is to be either flush with the concrete or slightly countersunk. This brass plug is designed to include the mark indicating the exact point of a station and the station numbering or designation. The base of the monument should be as large as, or larger than, the top. Where soil conditions are such that the use of a concrete monument is not practicable or desirable, 2-in. galvanized pipe with a cap screwed on for the upper end may be used as a monument.

A-9. "Site" and "Block" Plans to Show Control Stations.—The coordinate grid, horizontal control stations, and bench marks are to be shown in all project maps and plans. This is of special importance concerning "site" and "block" plans. This information is vital to speed and accuracy in the subsequent location survey work.

A-10. Horizontal and Vertical Control.—

(*a*) A system of monumented horizontal control traverses is to be established originating from, and closing upon, existing government control points. The error of closure of the traverse circuits shall not exceed (before adjustment) 1:10,000 for second-order traverse and shall not exceed 1:5,000 for third-order traverse. If the purpose of the survey is such that control of a lower order will suffice, the control points need not be monumented.

(*b*) In running traverse, a tape or chain should be used whose length has been determined by the Bureau of Standards when supported at each end and when supported throughout. The temperature of the tape during the chaining operation must be taken and proper temperature correction made. For proper slope chaining between "bucks," the elevation of each buck must be determined and chained lengths must be reduced to horizontal. Tension scales must be used to insure that the pull designated by the Bureau of Standards as the tension of accuracy is maintained. The tension scales should be tested and adjusted every three days. The springs of these scales weaken quickly and must be kept in adjustment. The field notes should show the tape number of the tape used. Should the tape be broken, another standardized tape must be used and the notes must show clearly the station number where this change was made.

(*c*) The transit used should be in first-class condition. It will be found economical to use an instrument, the least count of whose vernier is not greater than 20″, in which case the angle is to be doubled and the mean value used. If the transit used reads to the minute only, the mean value must be determined by six repetitions.

(*d*) Since control usually is to serve as the basis for all the subsequent surveys and maps needed on the project, the selection of its frequency and location must give due weight to this diverse need. In areas where there is to be detailed topographic mapping, and probable intensive development, the lines of control circuits should not be more than from 2,000 ft to 2,500 ft apart in one direction but may be of any convenient dimension in the other direction. Control points should not be more than from 800 ft to 1,000 ft apart along the line of the circuit and must be intervisable. Where topography is to be taken by the plane-table method, a layout of the plane-table sheets should first be made, and the control circuits, near the sheet border, should be so selected that the line may be platted on both sheets.

(*e*) Where control is to serve for general topographic surveys and for controlled mosaics, only, the control circuits may be much farther apart. These larger circuits may be cut up by stadia lines such as may be needed to maintain the specified map accuracy, where the control is for the combined plane-table aerial photography method of mapping. The control points should be at or near identifiable picture points, such as right-angle intersections of roads, fences, and streams. The control party should have copies of the contact prints, and mark upon these prints the identification of the control points as located. Transit-stadia lines run for this purpose should have an accuracy of closure on the primary control of 1:1,200.

(*f*) As nearly as possible, in the interest of speed and accuracy, control circuits should follow existing roads and easy grades but should avoid traffic interference. Control stations should be so selected as to be least subject to destruction from possible road widening or other construction. The monuments should be protected by two or more guard posts at least 3 ft high.

(*g*) Vertical control shall consist of levels run in circuits, the closure error of which shall not exceed $0.03 \times \sqrt{M}$ ft (M being the distance in miles). The elevation of each traverse station monument shall be determined. These points must be in circuit (turn points) and may not be side shots. Other permanent bench marks should be set at frequent intervals.

Section B. Specifications for General Topographic Maps

In addition to the general specifications applying to all topographic maps, the following specifications shall apply:

B-1. Map Scales.—General topographic maps may be at scales varying from 1 in. equals 400 ft to 1 in. equals 1,000 ft, and at contour intervals varying from 5 ft to 10 ft. The selection of the proper scale and contour interval is controlled by: The character of the terrain, the density of the culture, the size of the area, and any special "project" considerations. The map is to show the important physical features such as roads, railroads, streams, houses, fences, and cleared and timbered areas. Elevations are to be shown by contours and by spot elevations—the latter to be at road intersections, bridges, water surfaces, summits, and depressions.

B-2. Accuracy.—Distances from the nearest control point, of definitely recognizable points as platted on the map, shall not be in error by an amount to exceed 0.1 in. on the map scale. The contours shall portray the general topographic shapes, and elevations determined by interpolation from the contours shall not be in error by an amount exceeding one contour interval. Spot elevations shown in the map shall be correct within an amount, in feet, represented by one fourth of the contour interval.

B-3. Control.—A system of horizontal and vertical control must be established to regulate the accuracy of the map, as described for horizontal control in Section A-10. When the need for the map is immediate and speed is so urgent that a complete system of primary control cannot be waited for, a circuit of primary control, both horizontal and vertical, should be run. This should be cut up by circuits of secondary control which may be run by transit-stadia method. The primary circuits must be so selected as to control and keep the secondary circuits within the specified limits of map accuracy. These secondary circuits should originate from, and close upon, points located by the primary control. The error of closure should never exceed 1:1,000 and should average 1:1,200 or better. To maintain the specified map accuracy which limits error of position to 0:1 in. on map scale, no such secondary control circuit should be more than 7 miles long. Levels of the type known as "flying levels" should be run over the secondary control circuit, and the error of closure on the primary control must not exceed 0.3 ft.

B-4. Map Tests and Criteria of Acceptability.—To determine whether surveys and maps, procured under contract, comply with the specified accuracy requirements, they are to be tested either by check profiles or by area tests, or by both. The test surveys are to originate from, and close upon, control points. The test profile is to be not less than 3,000 ft long. The area tested shall be a strip not less than 600 ft wide (300 ft each side of a base or profile line) in which the location of contours within this area or elevations of selected points are determined. If 90% of the points tested comply with the accuracy specifications, the map shall be deemed acceptable. No map or map sheet shall be rejected unless two tests have been made. Each test is to be of a different area comprising the map or map sheet.

B-5. Map Drafting.—The map shall be drawn on sheets of the best quality drafting paper obtainable—well seasoned and double mounted. Otherwise, the sheets may be on the best quality well-seasoned single mounted paper. The drawings shall be drafted in one color only (jet black waterproof ink). All lines must be clear, sharp, and distinct. Lettering shall be neat and legible

and of a size to be readable in case of a 2 to 1 reduction. The inked sheets are to be copied photographically producing a film positive to scale. Prints may be made from these film positives by the ozalid or other dry reproduction processes. Reductions or enlargements to any desired scale may also be made from these positives.

Section C. Specifications for Detailed Topographic Maps

C-1. Map Scale.—Detailed topographic surveys at map scales vary from 1 in. equals 20 ft to 1 in. equals 200 ft, depending on the density of cultural features, character of terrain, and area involved. The contour interval is to be 1 ft or 2 ft.

C-2. Data to Be Shown.—The map sheets shall show: All control points and bench marks (with designating number and elevation), all roads, railroads, streams, fence lines, utilities, poles, isolated trees (12 in. or more in diameter), boundaries of timbered areas, rock ledges or boulders, wells, buildings, cemeteries, and any other physical data that will affect planning. In addition to the elevations shown by the contours, spot elevations shall be shown at all summits, bottoms of depressions, tops of banks, stream surfaces or water surfaces, road and railroad lines, breaks of grade, and at intersections and at bridges.

C-3. Accuracy.—Projection grids and control points shall be platted correctly to the nearest 0.01 in. on the map scale. The horizontal position of any definitely recognizable point shall be correct with reference to the nearest control point within 1/50 in. on the map scale. The contours shall express, faithfully, the relief detail and topographic shapes. In addition, elevations or profiles interpolated from the contours shall be correct within one half of the contour interval. Spot elevations shall be correct to the nearest 0.1 ft.

C-4. Map Drafting.—The map shall be drawn on sheets of the best quality drafting paper obtainable—well seasoned and double mounted. Otherwise, the sheets may be on the best quality well-seasoned single mounted paper. The drawings shall be drafted in one color only (jet black waterproof ink). All lines must be clear, sharp, and distinct. Lettering shall be neat and legible and of a size to be readable in case of a 2 to 1 reduction. The inked sheets are to be copied photographically producing a film positive to scale. Prints may be made from these film positives by the ozalid or other dry reproduction processes. Reductions or enlargements to any desired scale may also be made from these positives.

C-5. Map Tests and Criteria of Acceptability.—To determine whether surveys and maps, procured under contract, comply with the specified accuracy requirements, they are to be tested either by check profiles or by area tests, or by both. The test surveys are to originate from, and close upon, control points. The test profile is to be not less than 3,000 ft long. The area tested shall be a strip not less than 600 ft wide (300 ft each side of a base or profile line) in which the location of contours within this area or elevations of selected points are determined. If 90% of the points tested comply with the accuracy specifications, the map shall be deemed acceptable. No map or map sheet shall be rejected

unless two tests have been made. Each test is to be of a different area comprising the map or map sheet.

C-6. Errors Arising from Paper Distortion.—To minimize errors arising from paper distortion it is recommended that maps be drawn on metal mounted map sheets and that a method of reproduction direct from the map sheets be utilized. If situations should occur in which the use of tracing linen is unavoidable, errors arising from this use may be minimized if the following precautions are taken:

(*a*) Well-seasoned drafting paper of the best quality should be used.

(*b*) As soon as tracing linen is received, cut it into standard sheet sizes, and season it as well as possible by exposure to sunlight, moist, and dry weather; but do not hang the sheets up supported at one edge.

(*c*) Place the coordinate projection on both map sheets and tracing linen, before any other data are drawn on them. If this is done, the coordinate lines give a measure of the subsequent expansion or contraction which may be applied as a correction in plotting or scaling distance on the map, tracing, or blueprint.

(*d*) When tracing information from a map, keep those coordinate lines of the tracing linen in register with the identical lines of the original map, which surround the data being traced. In other words, work from the nearest coordinate lines, proportioning the discrepancy between the two coordinate grids in proportion to the distance from the grid lines. This can be done graphically by eye.

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DISCUSSIONS

COMPUTING COAGULANT REQUIREMENTS IN SLUDGE CONDITIONING

Discussion

BY A. L. GENTER

A. L. GENTER,[24] M. AM. SOC. C. E.[24a]—The discussions of this paper on the mathematical analysis of a practical and theoretical problem have originated in widely separated localities. In all efforts to understand natural phenomena, and to control them, both the practical and the theoretical are equally important. From the standpoints of presenting more statistical evidence of the validity and usefulness of Eqs. 5 and 6 to the designing engineer and operator and of critically examining the component parts of these equations—the discussions have been valuable contributions to the subject and are gratefully acknowledged.

Reply to them will be confined chiefly to the practical and theoretical phases of that part of Eqs. 5 and 6 which cannot be understood immediately in an intellectual manipulation of either equation. This difficulty refers to Eq. 4*b* which states mathematically that in addition to including the liquid alkalinity requirements of a sludge, measured in terms of water per unit weight of dry solids, the chemical requirement is a function of a constant multiplied by the weight of volatile matter per unit weight of ash present in the sludge.

Mr. McNamee agrees that the first part of Eqs. 5 and 6 (that part determined by Eq. 4*a*) is scientifically valid. He is logically cautious, however, about deriving equivalent validity from Eq. 4*b* and from the second part of Eqs. 5 and 6 without further evidence of supporting data. This caution arises chiefly from the difficulty of explaining the ratio of volatile to mineral matter in terms of the ratio of true organic matter to mineral matter. Mr. Fraschina agrees with this observation. As the writer has previously indicated (see heading, "Facts Revealed by Eq. 5") and will presently attempt to demonstrate, the volatile-ash ratio in these equations takes in considerably more territory in sewage treatment in general, and in sludge disposal in particular,

NOTE.—This paper by A. L. Genter was published in March, 1945, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: June, 1945, by Paul D. McNamee; and October, 1945, by D. L. McLean, and K. Fraschina; and December, 1945, by Walter H. Brown, Jr.

[24] Cons. Engr., Baltimore, Md.

[24a] Received by the Secretary March 11, 1946.

than has been generally contemplated in the examination of the effect of this ratio on coagulant consumption.

The mathematical language involved in the use of this ratio in Eqs. 4*b*, 5, and 6 does not necessarily imply measurement of organic matter itself. It does, however, measure the amount of sludge water present in a sludge per unit weight of sludge solids, and this is most important. The amount of volatile matter in a sludge evidently contains mostly organic and possibly some inorganic matter essential to the organic capillary structures—both of which ingredients are responsible for the polar-surface activities of the solid and its water-imbibing properties acting as capillary structures and colloidal aggregates. The ratio of volatile to ash, times some constant, therefore, measures these water-imbibing capacities instead of presenting any picture of the amount of true organic matter present in a sludge.

There is nothing novel in the idea that the amount of water in a sludge is some function of the volatile matter present. There is something new, however, in evolving mathematical tools for measuring this function. In discovering such tools, it seems to the writer that it is not so essential that the mineral-matter conversion factor in the volatile test be fairly constant, as it is that the analytical methods and data be kept on an "even keel" by closely following "standard-method" procedures. In fact, the data presented from all parts of North America, and the equations evolved therefrom, are a composite tribute to the reliable uniformity of standard scientific analytical procedure and the competence of analysts and observers.

Such equations cannot be derived without an ample supply of supporting data. The writer feels that this condition was met in the data presented in Table 1. He also feels that it is not so important that data be collected from seven or eight sludges as it is that they cover a wide range of treatment practices, reliable essential assays, and careful dosing operations. Nevertheless, the greater the number of data collected and correlated, the more reliable will be the average data used in deriving such equations. If a scientific hypothesis is sufficiently important, additional evidence of its truth should be sought. This is undoubtedly the meaning of Mr. McNamee's comment on this point and the writer thoroughly agrees with it. Therefore, he will attempt presently to supply ample additional evidence.

In addition to these observations, it should be noted that, unless the hypothesis geared into the mathematical machinery of such pioneering investigations is capable of confirming experimental averages already recorded and unless it becomes a guide in the investigation of new data and consequently becomes equally capable of predictions which observations later verify, the hypothesis is a museum piece instead of a useful scientific instrument.

At the time the data presented in Table 1 were subjected to the impartial machinery of mathematical correlation, the writer offered Eq. 5 and its more generalized expression Eq. 6 as instruments of prediction. The statistical data involved approximately 7,000,000 lb of sludge solids, 191,000 lb of ferric chloride, and 8,000 lb of alum.

In the succeeding five years, notwithstanding the influence of war emergencies on the nature of sewage treated, plant overloading, operation, changes in

volatile and alkalinity contents of the sludges, and additional recorded data, together with data from other plants where careful dosing is practiced, expanded the statistical data to considerably more than ten times the sludge solids and ferric chloride involved in Table 1. These additional data demonstrate that Eqs. 5 and 6 have served certain plants as reliable instruments of prediction and promise to continue to do so. This is made evident not only by the new data generously contributed and discussed by Messrs. McLean, Fraschina, and Brown but by considerable additional data collected from plants in Springfield, Mass., Hartford, Conn., and other places not tabulated and discussed.

All these data could be tabulated and summarized to indicate the validity of that part of the equations involving the so-called solids demand—that is, the constant 1.6 times the pounds of volatile matter per pound of ash—notwithstanding the great differences in the volatile and alkalinity contents listed in Tables 1, 2, 5, 6, and 7 and in the recent unpublished records from other plants. What could be demonstrated by such a comprehensive comparative tabulation can be made equally evident from the data relative to the Richmond-Sunset plant in San Francisco, Calif., in Tables 1 and 6.

Since the publication of the 1940–1941 data for that plant in Table 1 (line 5, Cols. 5 and 6), the volatile-ash ratio of the elutriated sludge has gradually increased from 1.74 to the 2.23 average of Table 6 (line 4, Col. 6). This material increase of more than 28% has taken place without any significant change in the yearly average value of the constant 1.6. Notwithstanding the causes of the increase in the volatile-ash ratio, incidental to the use of an overloaded single-stage digestion tank, occasional variations in the K-values of Table 6, and Mr. Fraschina's logical explanation of them, the 1940–1941 and 1944–1945 statistical averages for this plant have the same average value of 1.6 for K as do those listed for Winnipeg, Man., Canada, Hartford, Conn., Springfield, Mass., etc., and ultimately for Cranston, R. I. Since the publication of Table 7 relative to the Cranston plant, a private communication from Mr. Brown regarding the improved ferric feeder shows that the average K-value for the months of from June to December, 1945, also settled down to 1.6. That the value remained near this average all over North America can scarcely be thought mere coincidence.

Volatile-Ash Ratio and Sewage Treatment.—The writer has elsewhere emphasized[25] that sewage treatment, from plain sedimentation to final dewatering, is a system of correlated, progressive steps in removing undesired suspended and settleable solids from a large quantity of water. Everything in the system is connected in some definite manner with the amount of volatile matter in the solids being dewatered. This definite correlation may be traced from the effects of the volatile matter on the specific gravity of the solids in plain sedimentation (where gravity alone frequently accomplishes 99.96% of solids dewatering) to all the ever-increasing economic difficulties incidental to further dewatering.

[25] "Principles and Factors Influencing Vacuum Filtration of Sludge," by A. L. Genter, *Sewage Works Journal*, November, 1941, pp. 1164 and 1165.

In one of these final dewatering steps, investigations have disclosed the fact that the amount of solute-carrying water per pound of sludge solids is a function of a constant times the amount of volatile matter per pound of ash in the sludge—the inevitable result of conditions associated with this ratio in previous steps of the dewatering system. Therefore, the validity of the statement can be extended to: "Every step in this system is mathematically related to the volatile-ash ratio and some constant value."

The causes associated with this ratio will be found in the surface energy or polar nature of water related to polar groups and ions in the wetted solids. Water is the best solvent and ionizing agent and constitutes 99.9+% of sewage and from 90% to 98% of most sludges. Its solvent and hydration properties are due to the free surface energy of water which results from the polar character of its molecules (H − OH), "* * * because of which water acts as though it contains positive and negative electrical charges slightly separated from each other."[26] The polar nature of most solids submerged in water manifests itself by the presence of polar groups of molecules in the wetted solid surfaces (interfaces) having a strong solution tension or attraction for the polar-water molecules, as a result of which the molecules orient themselves toward the water:

> "The theory that molecules are oriented in surfaces and interfaces has a profound effect upon the development of surface chemistry, of colloid science and upon the theories of the internal structure of liquids,* * *."[26]

This theory also accounts for the surface charges of the wetted solids, for coagulation phenomena, and for all the water-imbibing properties of capillary gelatinous structures containing volatile matter and ash—namely, starches, agars, albumin, gelatin, pectins, zoogloeal masses, activated sludge floc, and all organic solids found in sewage which are subject to bacterial decomposition. Even hydrated mineral structures of a gelatinous nature that show volatile and ash contents by decompsoing at red heat owe their powers of hydration to the same polar forces—silicic acid gels and the flocculent precipitates of iron and aluminum hydroxide. Furthermore, most of the finely divided mineral solids found in sewage are attracted to water by polar groups such as silica, clays, oxides, and sulfides.

In capillary-active solid and semisolid structures common to most suspended and settleable sewage solids, the volatile and mineral ingredients are primarily responsible for the polar activities, hydration properties, and surface chemistry pertinent to the subject. In these structures the ingredients are related in some definite manner to other definite properties, such as:

(1) Specific gravity of sludge solids;
(2) Hydraulic subsiding rates or settleable solids;
(3) Capacity of sludge solids for imbibing and retaining large amounts of enmeshed water;
(4) Concentration of polar solutes in the imbibed and enmeshed water which necessarily has an influence on item (5);
(5) The chemical coagulant requirements; and
(6) Elastic properties of solids aggregates (compressibility of solids).

[26] "Surfaces of Solids in Science and Industry," by William Draper Harkins, *Science*, Vol. 102, September 21, 1945, pp. 293–294.

Item (6) concerns deformation under filtration pressure and has been fully discussed elsewhere.[27] Like solids, aqueous gels are characterized by definite elastic properties.[28] Although the influence of volatile matter on sludge compressibility in filtration is quite pronounced and remains to be mathematically investigated, it is altered advantageously by chemical coagulation, which, in items (4) and (5), has been mathematically related to the volatile-ash ratio.

In pushing the causes associated with this ratio in items (4) and (5) back a step to item (3), one encounters some fresh evidence of a most convincing nature on the general validity of this basic ratio. This evidence primarily involves referring to another field of organic colloids in order to relate the water-carrying power of solid gels to the amounts of volatile matter and ash in such gels.

Most gelatinous or amorphous aggregates of sludge solids are similar to gels, zoogloeal and otherwise. This fact becomes quite evident in relating hydration properties to the volatile matter and ash present. C. K. Tseng,[29] in his article on seaweed colloids (phycocolloids), shows that the various types of agars, in natural conditions, are calcium, magnesium, potassium, and sodium salts of various agar acids. The gel-forming properties of the various agars, and their allied phycocolloids, are functions of the ash contents, which is equivalent to stating that they are functions of the volatile and ash contents. The dried amorphous agar which scientists use (the standard agar of the bacteriologist) is prepared from seaweed species Gelidium, has an ash content of less than 5%, and contains 95% or more volatile matter. Such agars are:

> "* * * insoluble in cold but soluble in hot water, a one per cent neutral solution of which sets at 35° to 50° C to a firm gel melting at 80° to 100° C."

In other words, the solid gel of 19 to 1 volatile-ash ratio imbibes ninety-nine times its own weight in water to set to a solid gel that has a melting point near the boiling point of water.

Another seaweed colloid is carrageenin, which resembles agar in a general way, but has a much higher ash content (about 20%, or a volatile-ash ratio of 4 to 1) and requires concentrations of 3% or more to form solid gels containing about thirty times its own weight in imbibed water. These gels also melt at lower temperatures.

In any case, the Tseng article[29] clearly demonstrates that the power to imbibe water and to form firm gels, at ordinary temperatures, by different seaweed colloids is definitely a function of the volatile and ash contents of the different types listed. These colloids are believed to be chiefly cell-wall constituents of various seaweeds, comparable to the pectins of the land plants.

These pertinent facts were emphasized because the hydration and water-imbibing properties of such gels are remarkably similar to the disclosures relative to the water-imbibing and enmeshing properties of activated sludges

[27] "Principles and Factors Influencing Vacuum Filtration of Sludge," by A. L. Genter, *Sewage Works Journal*, November, 1941, pp. 1171, 1174, and 1177 to 1179, inclusive.

[28] "Colloid and Capillary Chemistry," by Herbert Freundlich, E. P. Dutton & Co., Inc., New York N. Y., 1922, pp. 716–719.

[29] "The Terminology of Seaweed Colloids," by C. K. Tseng, *Science*, Vol. 101, June 15, 1945, pp. 600 and 601.

made by Gail P. Edwards,[30] M. Am. Soc. C. E., as will become more evident through mathematical analysis of Mr. Edwards' disclosures. His pioneer studies relating to the effects of volatile matter on solids concentration in sedimentation tanks have deserved much more attention from the research engineer and scientific investigator than has heretofore been evidenced. They have been particularly valuable to the writer in the present investigation, not only because of the ample supply of statistical evidence on the influence of volatile matter on the water content of the sludges investigated, but also because of the wide range of volatile contents in the sludges. The percentages of volatile matter in the sludges investigated at the sewage treatment plants at Wards Island and Tallmans Island in New York, N. Y., ranged from 38 to 85.5, thus the volatile-to-ash ratios varied from 0.61 to 5.9.

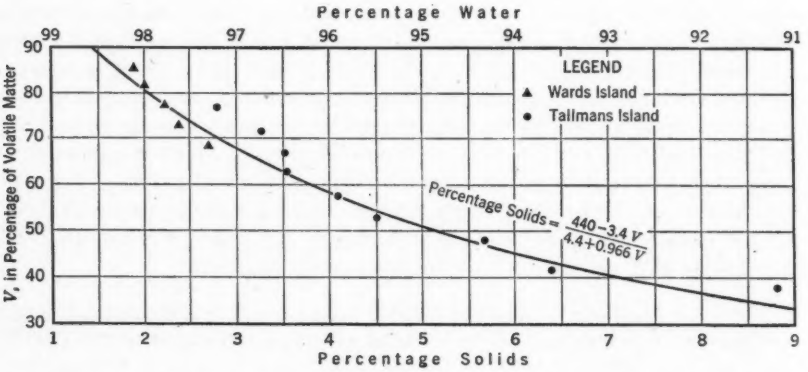

FIG. 5.—RELATION BETWEEN PERCENTAGES OF VOLATILE MATTER AND SOLIDS (ILLUSTRATING EQ. 13)

Fig. 5 illustrates the graph of the mathematical equation evolved by the writer from the original data published by Mr. Edwards.[31] Separate graphs are shown by Mr. Edwards for Wards Island and Tallmans Island sludges.[32] In Fig. 5, the full line is plotted from the writer's equation and accounts for both sludges. The various data-point averages for the two New York plants are indicated in Fig. 5 by black circles for Tallmans Island and by black triangles for Wards Island. The percentages of volatile matter are plotted as ordinate (y-axis) values, and the percentages of solids are plotted in ascending order, with the corresponding percentages of water in descending order as abscissa (x-axis) values.

Mr. Edwards' published data and graph averages for Tallmans Island indicate a curve of a hyperbolic type which (when the different y-axis and x-axis values are symbolized as V equals percentage of volatile matter, W equals percentage of water, and S equals percentage of solids in sludge) the writer

[30] "New Developments in Activated Sludge Plant Operation," by Gail P. Edwards, *Sewage Works Journal*, November, 1940, p. 1080.

[31] *Ibid.*, p. 1080, Tables I and II.

[32] *Ibid.*, p. 1081, Fig. 3.

evolves as:

$$W = \frac{100\ V}{4.4 + 0.966\ V} \quad \ldots\ldots (12)$$

and

$$S = \frac{440 - 3.4\ V}{4.4 + 0.966\ V} \quad \ldots\ldots (13)$$

Dividing Eq. 12 by Eq. 13 to determine the quantity of water per unit weight of solids (Eq. 1), a fact of major importance to this discussion is developed:

$$\frac{W}{S} = \frac{29.4\ V}{29.4 + 100 - V} \quad \ldots\ldots (14)$$

Because the value $100 - V$, as formerly stated in developing Eq. 4*b*, is nothing but the percentage of ash in the sludge, Eq. 14 clearly states that in these particular sludges:

$$\frac{\text{Water (\%)}}{\text{Solids (\%)}} = K' \frac{\text{Volatile (\%)}}{K' + \text{Ash (\%)}} \quad \ldots\ldots (15)$$

Eq. 15 is undoubtedly the general law expressing the amount of evaporable water per unit weight of dry solids in sludges containing volatile matter. It establishes the measurable influence of the volatile-ash ratio and some constant value on the water-carrying power per unit weight of sludge solids in the first stage of solids dewatering in sewage treatment (that is, in sedimentation) and confirms the validity of this influence in sludge dewatering by vacuum filtration.

Eq. 15 also clarifies the subject of the capacity of sludges to imbibe water. In the presence of the total water included in this equation, the ratio of volatile percentage to a constant plus the ash percentage is somewhat greater than the volatile-to-ash ratio itself. Since the ratios in the equation were determined from the sedimentation of wetted solids and include enmeshed water freely associated with the settled solid flocs, this additional constant amount indicates a water-imbibing or enmeshing factor (see sludge property item (3)). This factor will vary greatly with the polar or water-binding properties of the sludge solid structures—it will be highest with the water-enclosing, gelatinous structures of activated sludge floc, before and after digestion (also with other hydrated gel structures) and lowest with denser and less hydrated sludge solids. Processing suspended solids through activation and primary solids through digestion increases their value.

The average of several of the most prominent treatment plants shows K'-values of about 7 for primary raw sludges, 16 for digested primary, and 27 for waste activated and digested waste activated in plants having primary sedimentation with sludge activation. The actual observed K'-values for Tallmans Island average 26.2; and, for Wards Island, 31.5. The average values at the Southerly and Easterly plants at Cleveland, Ohio, for concentrated waste activated sludge fed to digestion and after digestion agree fairly well with the Tallmans Island values.

As the K'-value includes enmeshed water freely associated with the flocculent or hydrated structures of various sludge solids, the value will obviously

vary somewhat with the detention time in sedimentation and with sedimentation tank design. Without standards for these factors, the K'-value is best determined by evaporation. In any case it indicates the existence of water-imbibing and enmeshing properties resulting from the sludge-solid structure associated with the amount of volatile matter per unit weight of mineral matter present.

It must be noted that Eqs. 5 and 6 really express the sum of two conveniently separated coagulant-consuming reactions occurring in the same total amount of water per unit weight of solids as shown by Eq. 15. The quantity $\frac{K' V}{K' + 100 - V}$ in Eq. 15 could be used to represent the water-solids ratio in Eq. 6 and would then be related to the quantity $K \frac{V}{100 - V}$ of the solids demand in Eq. 6. When multiplied by the ferric chloride constant 1.08, the constant K' in Eq. 15 would be some multiple (n K) of the basic K in the solids demand. Notwithstanding the magnitude assigned to the values K and n K by practice and theory, the volatile-to-ash ratio in the sludge remains the essential factor in determining, in terms of sludge solids, the total water containing the solutes reacting with the added coagulants in free solution and in the wetted solid surfaces.

There is ample theoretical background for the foregoing disclosures provided by the buoyant effect of the lighter volatile fraction on the specific gravity of the sludge solids. The general statement found in the literature that the volume of a sludge (that is, the ratio of water to solids) depends mainly on water content and only slightly on the character of the solid matter can be quite misleading. The simple truth lies in reversing this statement; thus:

> "It is the capillary and polar characters of the wetted sludge particles and gelatinous aggregates of the sludge submerged in water that determine both the specific gravity of the sludge solids and the water content of a sludge."

Sufficient evidence has been supplied to demonstrate that the amount of water per unit weight of solids in a sludge is a function of the amount of volatile matter per unit weight of ash.

The influence of this important ratio on the specific gravity of sludge solids and their consequent gain or loss in hydraulic subsiding values are of prime importance to the general subjects of sedimentation and further sludge dewatering.

Among the factors controlling free settling rates of solids in water—particle size and structure and the frictional or viscous resistance of water to the falling solid—the gravitational pull on the solid is of prime importance. This is measured in terms of the gravity constant times the difference in the specific gravities of the solid and water. In centimeter-gram units, the difference indicates the apparent weight of the submerged solid. The apparent weight loss in water occurs because of the buoyant force of the water on the solid; and, in the plain sedimentation of sewage solids, the apparent weight loss depends on the volatile-ash ratio. This loss increases as the specific gravity of the sludge

solids (G_s) decreases. Since the specific gravity of the volatile matter (G_v) is so near that of water, it has a material influence on this loss or on the resulting buoyant force of the water on the sludge solids.

The weight loss of sludge solids in water is the sum of the losses of the volatile and mineral fractions or percentages in the solids—namely, with 100 grams of solids in water the sum of the cubic centimeters of water displaced and the apparent total weight loss in grams is $\frac{V}{G_v} + \frac{100 - V}{G_m}$, in which V is the percentage or grams of volatile matter; $100 - V$, the percentage or grams of ash present; and G_v and G_m, the respective specific gravities of the volatile and mineral matter.

In this sum of apparent losses that of the lighter volatile part is always $\frac{G_m V}{G_v (100 - V)}$ times greater than that of the mineral part; that is: $\frac{V}{G_v} \div \frac{100 - V}{G_m} = \frac{G_m V}{G_v (100 - V)}$. As the average values for G_m and G_v will be fairly constant in most plants, this ratio (R) of the weight loss of the volatile part to that of the mineral part of sludge solids may be expressed as:

$$R = K_2 \frac{\text{Volatile (\%)}}{\text{Ash (\%)}} \qquad (16)$$

in which K_2 is the ratio of the specific gravity of the mineral part of the solids to that of the volatile part.

With this constant in mind Eq. 16 simply measures the gain or loss in the buoyant power of water on the submerged sludge solids through the increase or decrease in the volatile-ash ratio of the sludge.

The ratio R of the buoyant force of water on the volatile part to that of the mineral part of a sludge is a straight-line function of the specific gravity constant K_2 times the volatile-ash ratio of the sludge. For example, with $G_m = 2.5$ and $G_v = 1$, K_2 equals 2.5 and the buoyant power of water on the volatile part of a sludge averaging 50% volatile matter (volatile-ash ratio 1 : 1) is 2.5 times greater than that on the mineral part. The buoyancy ratio doubles in a sludge having a 2 : 1 volatile-ash ratio (66.7% volatile : 33.3% ash) and quadruples in a sludge having a 4 : 1 volatile-ash ratio (80% volatile : 20% ash).

The buoyancy ratio therefore is one of the fundamental causes of the gain or loss in hydraulic subsiding rates of settleable solids and of the consequent gain or loss in the quantity of water with its solute per unit weight of sludge solids in the progressive dewatering steps involving Eqs. 5, 6, and 15.

Adding the imbibed and adsorbed water, previously associated with the polar nature of the volatile solids, will change the effective displacement volume of the volatile and mineral parts of a sludge and the buoyancy ratios expressed by Eq. 16. Thus, both the free and impeded settling rates and the final water content of the sludge will be changed. The final effect has already been accounted for in Eqs. 13, 14, and 15 and the discussion concerning them.

The plot of percentages of volatile matter against the specific gravity of sludge solids (G_s) determined by the usual equation $G_s = \frac{100}{\frac{V}{G_v} + \frac{100 - V}{G_m}}$ results

in a graph similar to Fig. 5 for the same volatile and solids range. However, the latter experimental graph shows the buoyant effects of imbibed and enclosed water in the settled sludge floc. Furthermore, experimental Eq. 13, when extended to a sludge containing solids of a hypothetical 100% volatile content, shows 1% solids and 99% water content. Although such an extension may lead to erroneous conclusions, the graphed values taken within the experimental range do indicate that at Tallmans Island the specific gravity of the volatile matter is slightly greater than that of water—probably 1.02 or 1.04—which denotes that some hydrated mineral matter later appearing in the ash is probably a part of the wetted volatile structure.

Although the analytical determination of volatile matter and ash in sewage treatment is not as scientifically exact as chemical determination of solutes in the sludge water, it is quite as exact and reliable as the current method of determining the amount of water and dry solids by evaporation. The measurements involving weighted average percentages of moisture and solids, volatile matter, and ash in all the dewatering steps are sufficiently accurate and informative when computed to the first decimal place. Therefore, further precision is not warranted in measuring weighted average percentages of solute and coagulant on solids that have been determined by evaporation. With this in mind, measurements of amounts of water per unit weight of sludge solids and of volatile matter per unit weight of ash are just as reliable yardsticks in Eqs. 5, 6, 15, and 16 as is the law of combining weights in Eqs. 5 and 6; and they have quite a reliable scientific background.

In these equations and those from which they are derived, the separation of a sludge into volatile and ash components is analytically convenient in relating and measuring the various properties of polar water and wetted polar volatile solids to successive solids dewatering steps. As noted before in discussing Eq. 15, the separation of sludge requirements for coagulating chemicals into an alkalinity demand, and added solids demand, is also expedient. The solids demand is altered by digestion and the alkalinity demand may be altered by elutriation previous to chemical conditioning. Nevertheless, both reactions involve the same solute ions in the same sludge water. In addition to the alkalinity reactions, there are the rather widely known oxidation-reduction reactions associated with the use of ferric chloride, which are conveniently associated with the carbon compounds, sulfides, etc., in the wetted solids demand. Probably other minor reactions are associated with this demand. However, because of the presence of calcium and magnesium in sludges, it is doubtful whether borates, silicates, and phosphates will be found in sufficient quantities as true solute ions to affect the coagulant consumption in average sludges significantly. The chief polar ions involved in the surface chemistry of coagulation will be the various bicarbonates.

Aside from the polar compounds found in the chemically and biochemically fouled water associated with wetted sludge solids, the surface chemistry of the polar solids holding this water and its solutes is active in coagulation. As described elsewhere,[33] the process of coagulating water-logged sludge solids is

[33] "Principles and Factors Influencing Vacuum Filtration of Sludge," by A. L. Genter, *Sewage Works Journal*, November, 1941, p. 1179.

similar, in principle, to the curdling of milk and depends on the chemical transformation of polar groups in the water-imbibing structures, with the result that these structures clump into solid aggregates having far less attraction for water.

Such clumping may also be termed a coarse flocculation process, but the rate of obtaining properly clumped solid aggregates in coagulating sewage sludges has only a limited likeness to the rate of flocculation common to the colloid chemical practice of coagulating rather pure sols dispersed in relatively pure water. The writer, therefore, doubts if the clumping rate in current vacuum filtration practice has any measurable effect on ferric chloride consumption. He does agree, however, that both the method of mixing coagulant with a sludge, and the duration of the mixing, are of material importance to coagulant economy. This has been emphasized elsewhere,[8,34] including the last paragraph of the "Summary."

In any case, the research involved in this paper and in its discussions should leave little doubt that the coagulant requirements of various sludges are functions of the solutes in the sludge water; and, what is most important, the amount of this water per unit weight of sludge solids is a function of the amount of volatile matter per unit weight of ash in the sludge. In presenting the foregoing investigations of the causes and consequences of this relationship from the initial stages of solids concentration in plain sedimentation to the final stages of the disposal of the solids, it is felt that the amount of evidence has left the problem less baffling.

[8] "Laboratory Control of the Operation of the Richmond-Sunset Sewage Treatment Plant," by K. Fraschina, *Sewage Works Journal*, January, 1940, pp. 95–96.

[34] "Principles and Factors Influencing Vacuum Filtration of Sludge," by A. L. Genter, *ibid.*, November, 1941, pp. 1187, 1188, and 1189.

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DISCUSSIONS

DESIGN OF ROADBEDS

Discussion

BY IRA B. MULLIS

IRA B. MULLIS,[43] ESQ.[43] —The science of roadbeds is being retarded by the use of too many empiricisms. Too many engineers have become engulfed in channels of procedure so deep as to obscure many of the simple laws of physics, thus beclouding the essentials of roadbed design. It was this situation that caused the writer to begin the paper with a review of the properties of gases, liquids, and solids which one discusser referred to as "academic." Therefore, for the sake of directing attention to these hidden causes of roadbed failure, the writer gladly accepts the criticism.

Attention is again directed to the effect of air at point A, and that of water at point B, in Fig. 1. The effect of these fluids on penetration resistance (load) is so apparent that no one familiar with stress-strain curves should overlook it. But they do! Common experience should tell one that for maximum resistance to weather and traffic the apex of the density-moisture curve should intersect the penetration resistance curve midway between points A and B.

Rock matter of which roadbeds are formed consists of minerals having usually a definite chemical composition and, if formed under favorable conditions, a certain characteristic atomic structure is expressed in its crystalline form and in other physical properties. Even clay, the most weathered and changed residue of the original minerals, consists largely of crystalline matter and hence has definite properties. Of these properties, specific gravity is an important one. The specific gravity of kaolin, according to E. S. Dana and W. E. Ford,[44] is from 2.60 to 2.63, a value not far removed from that of the feldspars. It should be stated, however, that certain of the less common members of the kaolin group may have a specific gravity as low as from 2.00 to 2.20, although the high silica content of most clays utilized in engineering work causes the value to range near 2.65. In the absence of definite information,

NOTE.—This paper by Ira B. Mullis was published in April, 1945, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: October, 1945, by Charles W. Britzius, J. T. L. McNew, and Gustavo Pérez Guerra; November, 1945, by Jacob Feld, and Jasper L. Stuckey; December, 1945, by Delbert L. Lacey; January, 1946, by René S. Pulido y Morales; and February, 1946, by K. B. Woods.

[43] Cons. Engr., Raleigh, N. C.

[43a] Received by the Secretary February 13, 1946.

[44] "Textbook of Mineralogy," by E. S. Dana and W. E. Ford, John Wiley & Sons, Inc., New York N. Y., 1932.

one should remember that a ±0.05 change produces a difference in pore space in most cases less than 4%. Another important property, particularly involving fine sediments such as silt and clay, is the critical point at which they become plastic. Other critical points of crystalline materials need not be discussed because of their lack of application to roadbeds.

Critical Moisture.—A very important critical point of roadbed material is the one at which a definite quantity of water causes a rigid body to change to a plastic one. Apparently no discussers were willing to contradict the value of this critical point although two denied its practicability. This phase will be discussed in a subsequent paragraph.

The "stability index" as defined in the paper (see heading, "Change of State" and the discussion of Table 3) is the ratio of the lower plastic limit to the moisture content. Since the date of first publication (April, 1945) research has been continued and the writer now (1946) desires to invert the ratio to read: The stability index is the ratio of the moisture content to the lower plastic limit. The latter definition is the one originally set up, but it was revised before publication to avoid an inverse relationship between the stability index and the stability stress (Cols. 10 and 11, Table 3). With this revision it is possible to combine the ratio of the moisture capacity to the critical moisture (lower plastic limit) with the stability index by multiplying the one by the other to give the "voids-moisture function" (Cols. 6, 7, 8, Table 10). By this

TABLE 10.—SERIES A; RELATIONSHIP BETWEEN MOISTURE, MOISTURE CAPACITY, CRITICAL MOISTURE, AND CAPACITY FOR LOAD

Specimen No.	Density (g per cm³)	Percentage Moisture			Moisture capacity Critical moisture [b]	Stability index	Voids-moisture function	Compressed strength (lb per in.²)
		Capacity [a]	Content	Saturation				
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
1	1.40	33.6	28.4	84.4	1.33	1.13	1.50	40
2	1.46	30.7	28.4	92.5	1.22	1.13	1.38	45
3	1.43	32.2	25.6	79.5	1.28	1.02	1.31	60
4	1.52	28.0	25.6	91.4	1.11	1.02	1.13	75
5	1.54	27.2	25.6	94.0	1.08	1.02	1.10	90
6	1.51	28.5	24.3	85.2	1.13	0.96	1.08	95
7	1.57	26.0	24.3	93.4	1.03	0.96	0.99	100
8	1.51	28.5	19.5	68.4	1.13	0.77	0.87	142
9	1.56	26.4	19.5	73.7	1.05	0.77	0.81	170
10	1.60	24.7	19.5	78.9	0.98	0.77	0.75	200

[a] The specific gravity assumed by the writer for computing moisture capacity is 2.65. [b] The recorded critical moisture (lower plastic limit) is 25.2%; the optimum is 27.7%.

combination it will be seen that an inverse relationship is found to exist between the voids-moisture function and the compressive strength of the cylinders of earth tested (Cols. 8 and 9). This relationship is to be expected as it is apparent that the actual water content and the pore volume also should affect strength in an inverse measure. The values in Cols. 8 and 9, Table 10 (from basic data presented by Henry C. Porter, M. Am. Soc. C. E.[45]), are shown graph-

[45] "Effect of Moisture on Compacted Soils Revealed by Laboratory Tests," by Henry C. Porter, *Engineering News-Record*, August 23, 1945, p. 101

ically in Fig. 5. The characteristic break in the curve is due, no doubt, to the effect of critical moisture that occurs between 0.90 and 1.00. Similar data are shown in Table 11 with like relationships shown between Cols. 12 and 13. In this table no stability index is shown. The explanation for this may be found in Col. 10 with values slightly higher than those in Col. 9 obtained at estimated saturation. This shows that actual moisture content (Col. 10, Table 11) is also

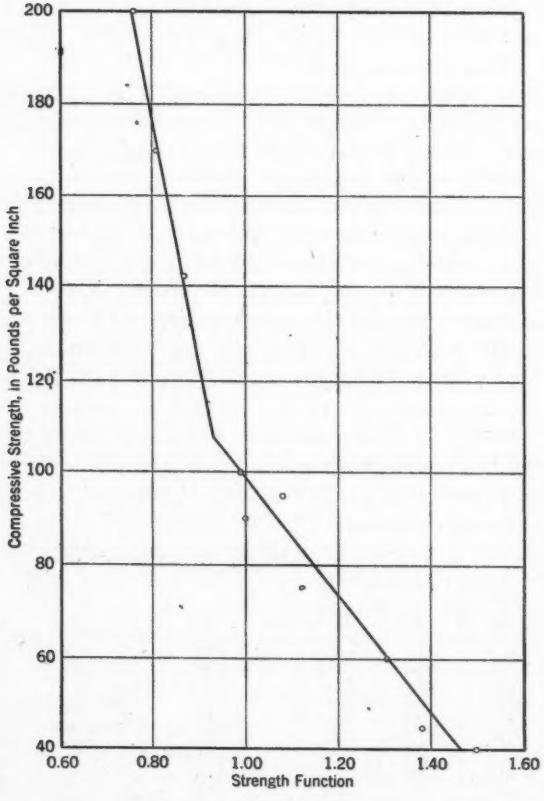

FIG. 5

practical saturation; hence, the voids-moisture function is equal to Col. 11 squared. Again the voids-moisture function follows the compressive strength values in inverse order without exception. These values (from basic data presented by Mr. Porter[45]) are shown graphically in Fig. 6, the break in the curve being somewhat in excess of the voids-moisture function 1.00. The extent to which this deviation may occur is not known.

Values in Table 12 show certain physical conditions found in a survey of roadbed conditions in the State of Iowa. The table is considered as being representative of roadbeds in that state that did, and those that did not, have

TABLE 11.—SEREIS B; RELATIONSHIP BETWEEN MOISTURE, MOISTURE CAPACITY, CRITICAL MOISTURE, AND CAPACITY FOR LOAD

Specimen No.	Condition at Start				Condition on 63d Day			Moisture at estimated saturation (%)	Condition at Final Test			Compressive strength (lb per sq in.)
	Density (g per cm³)	Percentage Moisture			Density (g per cm³)	Moisture (%)			Actual moisture content	Ratio[b]	Voids-moisture function	
		Capacity[a]	Content	Saturation		Content	Increased capacity					
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)
1	1.51	28.5	19.5	67.4	1.41	33.2	16.4	29.7	29.7	1.18	1.39	15
2	1.59	25.2	18.6	73.7	1.48	29.9	18.7	27.2	27.6	1.09	1.19	19
3	1.61	24.3	19.3	79.0	1.50	28.9	19.0	26.1	27.0	1.07	1.15	23
4	1.63	23.6	18.6	78.8	1.51	28.5	20.9	26.1	26.8	1.06	1.12	23
5	1.62	24.0	18.7	77.9	1.50	28.9	20.5	26.3	26.9	1.07	1.15	23
6	1.65	22.8	17.6	77.1	1.54	27.2	19.3	24.3	25.2	1.00	1.00	29
7	1.63	23.6	13.9	58.9	1.44	31.7	34.3	29.1	30.0	1.19	1.42	11
8	1.61	24.3	11.7	48.1	1.35	36.4	49.9	32.8	33.6	1.33	1.77	5

[a] The specific gravity assumed by the writer for computing moisture capacity is 2.65. [b] In the ratio, $\frac{\text{moisture capacity}}{\text{critical moisture}}$, the recorded critical moisture (lower plastic limit) is 25.2%; the optimum moisture is 27.7%.

TABLE 12.—SUMMARY OF MEAN VALUE OF PROJECTS IN THE STATE OF IOWA

Project	Absolute density	Cut density	Cut moisture	Pore capacity	Pore saturation	Liquid limit	Plastic limit	Ratio[a]	Surface thickness	County	No. of tests
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
(a) FROST-BOIL AREAS											
FA358	2.69	1.61	21.9	25.2	86	33	19	1.30	0.6	Delaware	18
FA173	2.68	1.48	25.2	30.4	83	47	22	1.37	0.2	Adair	24
FA60AB	2.71	1.58	21.4	26.5	80	44	19	1.38	0.2	Van Buren	7
FA134	2.66	1.55	23.4	27.4	87	40	20	1.35	0.7	Worth	6
FA851B1[b]	2.68	1.52	20.6	28.7	72	36	21	1.42	0.3	Webster	6
FA822	2.60	1.34	28.4	36.0	79	43	19	1.89	0.2	Tama	6
FA791A[b]	2.69	1.48	25.5	30.4	84	35	20	1.51	0.3	Marshall	6
FA7	2.68	1.51	25.0	28.9	87	33	19	1.52	0.2	Marion	1
FN544	2.66	1.49	26.1	29.7	87	37	19	1.56	1.1	Mahaska	5
FA873	2.68	1.51	26.6	29.0	92	39	21	1.38	0.3	Wapello	3
FA835C1	2.69	1.33	27.9	37.8	74	37	23	1.64	0.1	Lyon	6
P128B	2.70	1.56	25.2	27.4	92	47	21	1.28	0.2	Keokuk	6
FN234[b]	2.68	1.52	23.1	29.7	81	29	19	1.50	0.5	Jones	9
FA765[b]	2.68	1.51	23.1	29.3	79	32	18	1.63	0.5	Johnson	6
FA764B	2.69	1.50	26.9	29.7	91	39	22	1.36	0.2	Jasper	8
Means[c]	(2.68)	(1.50)	(24.7)	(29.7)	(84)	(38)	(20)	(1.49)	(0.4)		(117)
(b) NON-FROST-BOIL AREAS											
FA134	2.70	1.78	16.8	19.3	88	38	18	1.07	0.8	Worth	2
FA851B1[b]	2.68	1.51	21.9	28.9	76	36	20	1.44	0.2	Webster	1
FA791A[b]	2.71	1.85	15.2	17.0	89	32	15	1.16	0.2	Marshall	6
FA7[b]	2.71	1.60	21.8	25.6	85	50	19	1.32	0.2	Marion	3
FA765[b]	2.69	1.65	16.7	24.1	73	35	18	1.34	0.2	Johnson	8
FA764B[b]	2.72	1.76	15.7	19.5	79	43	19	1.05	0.2	Jasper	6
Means[c]	(2.70)	(1.69)	(18.0)	(22.4)	(82)	(39)	(18)	(1.23)	(0.2)		(26)

[a] Ratio of Col. 5 to Col. 8. [b] Part of project. [c] Grand average means; in Table 12(*a*), of fifteen items, and in Table 12(*b*), of six items.

frost boils. Col. 11, Table 12, shows a mean ratio of pore capacity to the plastic limit of 1.49 in frost-boil roadbeds and of 1.23 in those that did not have frost boils. The mean thickness of gravel on frost-boil roadbeds was found to be 0.4 ft and in those without frost boils only 0.2 ft. At the time the writer computed the data in Col. 10, he had not developed the term "voids-moisture function" although he has since computed the mean voids-moisture function of the frost-boil roadbeds in this survey and finds they have a value of 1.82. Those without frost boils have a value of 1.24. Therefore, by the use of the voids-moisture function, the relative strengths of the two conditions are probably more truly indicated.

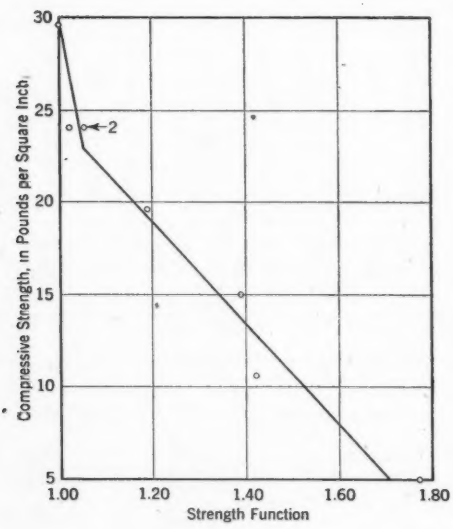

FIG. 6

Because of the lack of adequate experience in the use of the voids-moisture function, the writer hesitates to compare the values obtained from one material with those of another. He will suggest, however, that the frost-boil voids-moisture function of 1.82 be compared with specimen No. 8, Table 11, having a voids-moisture function of 1.77 and a strength of 5 lb per sq in. In a like manner, the non-frost-boil strength of 1.24 may be compared with specimen No. 2, Table 11, having a voids-moisture function of 1.19 and a strength of 19 lb per sq in. Again it may be compared with specimen No. 3 or specimen No. 4, Table 10, with a voids-moisture function of 1.31 and a strength of about 65 lb per sq in. This problem is one that seems to offer promise of success to the research engineer although additional research is needed.

Density.—The most striking manifestation of the density-strength relationship can best be shown by reference to Fig. 7, the rock cycle. This figure is nearly identical to one by Carey Croneis and William C. Krumbein[46] (see Richard M. Field,[47] and C. K. Leith and W. J. Mead,[48] Affiliate, Am. Soc. C.E.) but is modified by the insertion of the point at which the identity of the immediately preceding rock or sediment is lost. This loss of identity is the method used by the Committee on Sedimentation, Division of Geology and Geography, National Research Council,[49] as well as by William H. Twenhofel,[50]

[46] "Down to Earth," by Carey Croneis and William C. Krumbein, The Univ. of Chicago Press, Chicago, Ill., 1936, p. 212.

[47] "An Outline of Principles of Geology," by Richard M. Field, Barnes and Noble, Inc., New York, N. Y., 1938, p. 12.

[48] "Metamorphic Geology," by C. K. Leith and W. J. Mead, Henry Holt and Co., New York, N. Y., 15, Plate I.

[49] "Treatise on Sedimentation," by William H. Twenhofel, The Williams and Wilkins Co., Baltimore, Md., 1932, p. xxvii.

[50] "Principles of Sedimentation," by William H. Twenhofel, McGraw-Hill Book Co., Inc., New York, N. Y., 1939, p. 2.

for differentiating the several forms of rock matter. Several states or phases of rock formation are shown in Fig. 7. Fig. 7 includes all the major stages of the cycle, including magmas, igneous rocks, unconsolidated sediments, consolidated sediments, and metamorphic rocks. In each stage certain types of rock repre-

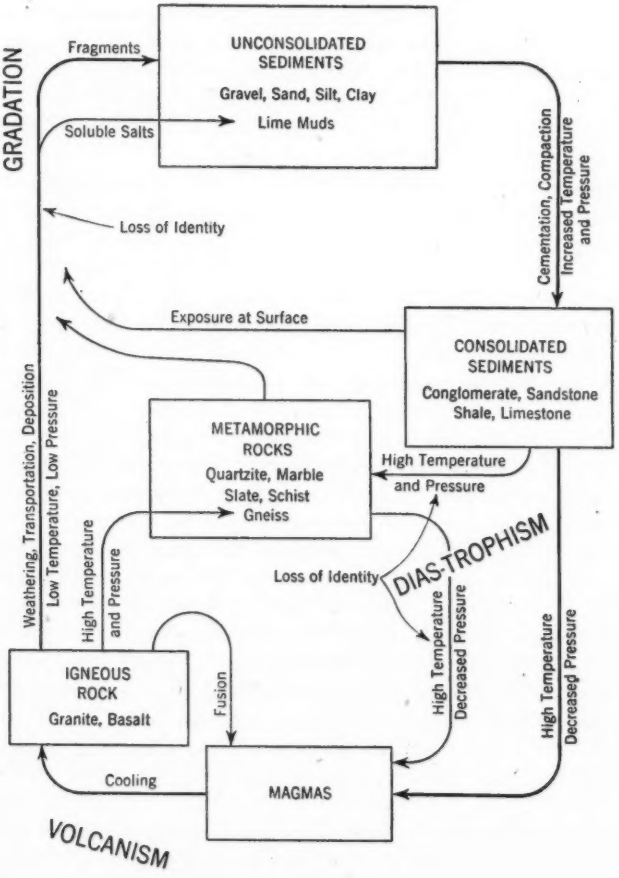

FIG. 7.—ROCK CYCLE

senting important members belonging to it are shown. The several stages are connected by arrows showing the rock trend under given conditions of environment. Attention is directed to the term, "loss of identity," designated as the practical point at which one stage of the rock cycle passes into the next one.

The strongest and most weather-resistant rocks, such as granite, have very small pore spaces and hence absorb very little water (Fig. 7). Of course weathering, including temperature changes, eventually enlarges the pore spaces and at the same time causes them to lose part of their capacity to resist loads

and weather. In the case of granites, for example, when weathering has caused the feldspar minerals to change to clay, the resultant material is a mixture of clay and sand (quartz). According to the authorities cited, the granite is said to have lost its identity and therefore becomes unconsolidated sediment, Fig. 7. If the parent granite is fine-grained, the resultant sediment will consist of fine sand, silt, mica, and clay. This unconsolidated sediment, whether wet or dry, will have much more pore space and hence can offer little resistance to loads and weather. The specific gravity of the parent granite was near 2.65 and that of the sediment is nearly identical. The outstanding change observed by the road engineer is the increase in pore space and attendant loss of resistance to loads and weather.

If a road is proposed across a belt of sediment derived from granite, the major problem of the engineer is the consolidation of the sediment to a measure that will not allow it to become plastic when fully saturated. The ideal toward which he should strive is to reduce the pore spaces to the measure of those in the parent rock, although the ideal is unattainable. Whether the proposed road is to be paved or used as an earth road, means must be found to seal the pores against the entrance of water in critical amounts or failure will follow sooner or later. How can this be accomplished? Returning to Fig. 7 one finds that nature converts unconsolidated sediments into more resistant consolidated sediments through cementation and compaction. Of course, the first step that should be made is to determine the plastic limit, the natural boundary between the rigid and plastic states. The second step should determine the possibility of consolidating the sediment to a measure that pore spaces will not receive water in critical amounts. To be successful, the engineer must determine the best method of converting certain unconsolidated sediments into consolidated equivalents. He must also know how nature converts gravel aggregate into gravel conglomerate, sand into sandstone, silt into siltstone, and clay particles into claystone. This is imperative if he is to build resistant roads. Moreover, this natural law applies to the making of Portland cement concrete, bituminous concrete, sheet asphalt, and sand-clay roads as well.

The application of data such as those shown in Table 10 is recommended as a means for analyzing roadbed problems and pointing the way to successful solution.

The values obtained on a homogeneous sample of clay in Table 10 computed from basic data by Mr. Porter[45] show the effect of density and water content on compressive strength. Table 10 also shows why the density-strength relationship is not always constant. For example, compare specimens Nos. 2 and 3, 5 and 6, also 7 and 8, in Table 10. Note the constancy of relationship between Cols. 8 and 9. Similar comments are also applicable to values in Table 11 computed from another series of tests by Mr. Porter and described in the same paper.

Table 12 shows the effect of density (Col. 3) and hence pore capacity (Col. 5) of roadbeds having frost boils and those without them. Although the mean percentage of pore saturation (Col. 6) in frost-boil roadbeds and the mean percentage of pore saturation in non-frost-boil roadbeds are nearly identical, one may observe the effect of low porosity (Col. 5) in the prevention of failure.

The part of Table 9 on this phase of the subject, in Mr. Lacey's discussion, shows striking contrasts between the modulus of deformation of like specimens of the higher densities (less wet) and that of those with the lower densities (wetter). In an effort to make the data as concise and conclusive as possible, the writer has recomputed the pore capacity in each case as a ratio of the lower plastic limit and presents it as Table 13. Without a single exception the re-

TABLE 13.—TRIAXIAL SHEAR TESTS (SUMMARY OF PART OF TABLE 9)

Sample No.	Test 1; Density Lower		Test 2; Density Higher		Col. 4 / Col. 2	Col. 5 / Col. 3
	Pore capacity / Lower plastic limit	Modulus value	Pore capacity / Lower plastic limit	Modulus value		
(1)	(2)	(3)	(4)	(5)	(6)	(7)
212	1.61	550	1.20	2,300	0.745	4.17
18	1.68	335	0.96	1,025	0.570	3.06
21*	1.12	470	0.80	4,755	0.715	10.12
44*	1.03	445	0.76	1,657	0.738	3.72
64*	1.03	1,990	0.61	3,195	0.593	1.60
72	1.53	600	1.16	2,655	0.759	4.41
84	1.80	611	1.26	2,680	0.700	4.37
130	1.17	1,095	0.96	1,385	0.821	1.26
146	1.37	336	0.98	2,300	0.716	6.84
160	1.24	1,280	1.12	3,720	0.914	2.89
Mean	....	....	....	....	(0.727)	(4.24)

* The four samples indicated in Col. 1 were remolded in the laboratory. The remaining sixteen were obtained in an undisturbed condition from the existing roadbed.

duction of the ratio (Cols. 2 and 4) causes a considerable increase in the modulus value (Cols. 3 and 5). In the case of sample No. 21 a reduction of about 70% of the modulus increases the strength about ten times. No doubt it would be very enlightening if one might have similar test data on identical samples compacted to produce pore capacities ranging from small to large values. Table 9 shows several cases in which an increase in density has produced a higher modulus even when the water content remains the same or is higher.

Messrs. McNew, Guerra, Feld, and Stuckey also support this method of approach in obtaining stability of roadbeds. Although Messrs. Britzius and Woods recognize the need of compaction, both seem to regard the standard proposed as impracticable although neither one offers any data in support of his belief. Apparently both base their opinions on the so-called Proctor density method (A.S.T.M. D 698–42 T and A.A.S.H.O. T 99-38) as their standard of compaction used in conjunction with light rollers of comparable effectiveness.

Mr. Britzius states that the writer's claim, that sediments compacted to low air voids are resistant to frost action, is misleading. Table 12 offers specific evidence in support of the value of low voids; and Tables 10 and 11 contain evidence showing that since water content decreases with pore space, less damage through frost action should follow.

Mr. Woods states that it is impracticable, if not impossible, to compact certain clays to a water-capacity value equal to 70% of their plastic limit; he is no doubt thinking in terms of light-weight rollers and not of ones with weights suggested by the writer. Although the writer does not claim that certain clays,

such as China clay and certain so-called pipe clays, can be compacted to prevent them from becoming plastic, he does insist that the use of excessive quantities of water, required to soften tough clays to bring them within the compaction range of light rollers, is a fallacy. This statement is proved when wide shrinkage cracks develop through drying after such light roller operation. Certainly one should expect roadbeds to be poor with pore spaces indicated in the last five tests in Table 5.

Through oversight, the writer failed to account for approximately 35% of coarse granular material in the first item in Table 5, referred to by Mr. Woods. This fact, of course, lowers the critical moisture in the mixture to perhaps 10%. On the basis proposed by the writer, the mixture should have a capacity of about 7%, although it is essential that the critical moisture point of mixtures be tested by the stress-strain method. Certain clays have been found with a density of 2.10 and experience teaches the writer that they are satisfactory when surfaced. It was this type of material the writer had in mind.

Compaction and Moisture Standards.—In 1933 R. R. Proctor[8] published a standard of compaction for earth dams which was intended to cause a body of earth to offer a needle resistance of not less than 300 lb per sq in. when saturated, and to restrict the percolation rate of water to 0.3 ft per yr. The quality of the work was measured by the capacity of the earth to support wheel loads and to resist the penetration of water. The weight of the roller and the number of its passes were such as to produce the required resistance to load and water. Density was intended only as a means for obtaining adequate resistance.

Some time after the introduction of this method, certain engineers undertook to improve on the Proctor method by applying a certain number of foot-pounds of energy to earth using the mold and rammer designed by Mr. Proctor. Whatever density this procedure produced was considered satisfactory by them. The method has been standardized by the American Association of State Highway Officials (T 99-38). It has also been adopted by the American Society for Testing Materials as a tentative standard (D 698-42 T).

Like other materials, clays and other roadbed materials offer varying resistance to compaction or shaping. China clay or kaolin, being weak, offers little resistance, although gumbo and adobe offer much resistance to change of shape unless quite wet. Of course, all clays are brittle when dry but tend to become tough when the ideal proportion of water is absorbed. This being true, the tougher the clay the greater will be the quantity of water required to deform a clay under a small force. Since the deforming force in A.S.T.M. D 698-42 T is only $5\frac{1}{2}$ ft-lb applied seventy-five times, it follows that the tougher the clay the greater will be the proportion of water required to soften it to the point where $5\frac{1}{2}$ ft-lb will eliminate the maximum percentage of air space.

Table 14 substantiates this point.[51] Col. 3 shows that the material represented by curve No. 1 is the toughest as it requires the largest percentage of

[8] "Fundamental Principles of Soil Compaction," by R. R. Proctor, *Engineering News-Record*, August 31, 1933, p. 246; September 21, 1933, p. 350; and September 28, 1933, p. 374.

[51] "Compaction of Subgrades and Embankments," Highway Research Board, National Research Council, August, 1945, p. 10.

water to soften it within the range of the rammer. Optimum moisture, of course, decreases constantly throughout the series. The density value follows in inverse order; but the percentage of saturation of pore spaces remains near 90% in every test, with a maximum deviation of 8.2%.

TABLE 14.—MOISTURE-DENSITY RELATIONS

Curve No.	Density		Optimum moisture (%)	Moisture capacity[a] (%)	Col. 3 / Col. 4	Col. 4 less Col. 3
	Lb per ft³ (1)	g per cm³ (2)	(3)	(4)	(5)	(6)
1	87.2	1.40	30.0	33.7	89.0	3.7
2	90.7	1.45	26.6	31.2	85.2	4.6
3	92.1	1.48	26.0	29.8	87.2	3.8
4	94.5	1.51	24.8	28.4	87.2	3.6
5	97.5	1.56	23.3	26.4	88.2	3.1
6	99.5	1.60	22.2	24.8	89.5	2.6
7	102.4	1.64	20.5	23.2	88.3	2.7
8	104.4	1.68	19.7	21.7	90.6	2.0
9	106.9	1.71	18.4	20.1	91.5	1.7
10	109.4	1.76	17.2	19.1	90.0	1.9
11	111.7	1.79	15.9	18.1	87.8	2.2
12	114.2	1.83	14.8	16.9	87.5	2.1
13	117.2	1.88	13.6	15.4	88.2	1.8
14	119.3	1.92	13.0	14.3	90.8	1.3
15	121.5	1.95	12.0	13.5	88.8	1.5
16	124.7	2.00	11.4	12.2	93.4	0.8
17	126.8	2.03	10.6	11.5	92.1	0.9
18	128.9	2.06	9.3	10.8	86.1	1.5
19	131.0	2.10	9.0	9.9	90.8	0.9
Mean	....	....	....	....	(89.1)	(2.2)
Maximum	....	....	....	....	(93.4)	(4.6)
Minimum	....	....	....	....	(85.2)	(0.8)

[a] For computing moisture capacity a specific gravity of 2.65 is assumed.

According to the facts presented and the experience cited in the paper, roadbeds constructed with densities such as those in the upper third of Table 14 should be expected to offer little resistance to loads or the penetration of water.

With adequate means for compacting the upper third of the table, it probably could have been compacted to densities and moisture contents shown in the lower third of the table.

To expect to produce adequate consolidation in a mass of an unknown clay by means of applying a given amount of energy is like supplying a given quantity of fuel to an unknown steam engine or gas motor and assuming the delivery of a definite amount of power from the shaft. Of course, this is a grave fallacy that cannot be condoned. The effectiveness of consolidation can be measured only in terms of the resistance it offers to loads and weather.

In an effort to uncover as much fact as possible from Table 8, the writer has recomputed the data with results as shown in Table 15. For these computations the specific gravity of the sediment is 2.62 and the lower plastic limit is 19. With a density of 1.51 the water capacity is nearly 1.5 times the critical moisture (19%) and hence is of low bearing capacity. The pore space is so great that little swell occurred when compacted with only 12% moisture (42.8% saturation). Obviously, air content would be displaced if water

entered and little swell followed. With a density of 1.85 and a moisture content of 14% (88% saturation) swell amounted to only 1.5% because the pore spaces were already so nearly saturated that little additional moisture was absorbed. The writer's interpretation of these and other similar data seems to indicate that for minimum volume change pore spaces should be reduced to a minimum, should be from 85% to 90% filled with moisture, and evaporation should be restricted by means of an adequate surface.

TABLE 15.—EFFECT, ON THE TOTAL SWELL, OF DENSITY AND THE PERCENTAGE OF SATURATION AT THE BEGINNING OF THE SWELL TEST (RECOMPUTATION OF TABLE 8; "SAT." DENOTES "SATURATION" AND "SW." DENOTES "SWELL")

Compacted density (g per cm³)	Water capacity (%)	Percentage Moisture When Compacted									
		10		12		14		16		18	
		Sat.	Sw.	Sat.	Sw.	Sat.	Sw.	Sat.	Sw.	Sat.	Sw.
1.51	28.0	35.7	1.0	42.8	3.2	50.0	2.6	57.1	1.6	64.2	....
1.60	24.3	41.1	5.5	49.3	3.6	57.6	2.6	65.8	1.8	74.0	1.2
1.70	20.6	48.5	5.7	58.2	4.4	67.9	2.6	77.6	1.6	87.2	0.7
1.80	17.4	57.5	5.6	68.8	4.1	80.4	2.5	91.9	1.4	....	0.0
1.85	15.9	62.8	5.5	75.5	3.6	88.0	1.5	....	0.8	....	....

Economy of Consolidation.—Objections to raising the standard of consolidation seem to be based on the belief that it will be impracticable or too expensive, although no cost estimate has been submitted by the discussers. The writer found that prior to 1940 many materials could be consolidated to adequate densities at a cost of from 4¢ to 5¢ per cu yd. Cost data published under the auspices of the Tennessee Valley Authority[52] show a cost usually less than 5¢ per cu yd for sheepsfoot rollers exerting from 300 to 400 lb per sq in.; 4¢ per cu yd is equivalent to a cost of 1⅓¢ for rolling 1 sq yd 1 ft thick. A cost of 10¢ per cu yd is only 3⅓¢ per sq yd 1 ft thick.

In lieu of high consolidation two discussers would stabilize with aggregates; but it should be remembered that aggregates must be placed on firm roadbeds to avoid potholes or other types of failure. Moreover, it is not economical to import selected material until local roadbed material has been rendered as efficient as may be economically possible. For example, it would not be economical to haul pit-run gravel costing 60¢ per cu yd to be placed 4 in. thick at 4.4¢ per sq yd when a square yard could be rendered stable for surfacing by rolling it 1 ft thick at a cost of 1⅓¢ per sq yd. Also, if one consolidated a road to a thickness of 1 ft at a cost of 1⅓¢ and then placed only 2 in. of gravel on it at a cost of 2.2¢, the total cost would be only 3½¢ and this of course would be much more valuable than the entire 4 in. of gravel placed on a less compact roadbed. When aggregates cost considerably more than a dollar per ton (as they usually

[52] "Granular Stabilized Base Construction of Access and Relocation Roads, by Tennessee Valley Authority," by F. W. Webster and F. H. Kellogg, *Proceedings*, Highway Research Board, Vol. 24, 1944, p. 483.

do), the economy of roadbed consolidation produces a striking contrast to usual procedure.

The reader is again directed to Table 12 showing the effect of roadbed compaction on the possible reduction of gravel thickness. Col. 5 shows a mean pore capacity of 29.7% in the frost-boil roadbeds with a mean gravel thickness of 0.4 ft (Col. 10). In contrast, the pore capacity in non-frost-boil roadbeds was 22.4% and did not fail with only 0.2 ft of gravel surface. As far as could be determined, no difference other than porosity existed. This survey seems to indicate that non-frost-boil roadbeds with a mean pore capacity of about 75% of that of frost-boil roadbeds remained stable with only one-half the gravel thickness, whereas frost-boil roadbeds failed. It is apparent, therefore, that a 25% reduction in pore capacity was of greater value than a 0.2-ft increase in thickness of gravel surfacing. Indeed if the cost of rolling should be as much as 15¢ per cu yd (5¢ per sq yd 1 ft thick) and the cost of crushed stone $1.40 per cu yd (7.8¢ per sq yd 4 in. thick)—a total of 12.8¢ per sq yd—this procedure would be more economical than spending the entire 12.8¢ for 6½-in. stone alone. On the basis of past experience, including the survey shown by Table 12, if the writer had only 8.9¢ per sq yd available, he would appropriate 5¢ for consolidation and the remaining 3.9¢ for a 2-in. crushed stone surface.

Summary.—As a summary the following conclusions are offered:

1. The change from the plastic to the rigid state is indicated by rupture.
2. The point of rupture in fine sediments, as determined by the Atterberg and stress-strain methods, are nearly identical (Cols. 5 and 6, Table 1).
3. The point of rupture is affected more by moisture than by load (Cols. 1, 2, 3, and 4, Table 1).
4. The apex of the ideal density curve of a roadbed should intersect the penetration resistance curve midway between points A and B, Fig. 1.
5. Masses of earth compacted, as described in the paper under the heading, "Change of State: Consolidated Roadbeds," and in Table 6, absorb little additional water; hence, they remain stable.
6. The plasticity index is of very limited use in predicting resistance to impact or weather (Figs. 3 and 4).
7. The "voids-moisture function" appears to offer splendid possibility in predicting the capability of roadbeds to resist load and weather (Tables 10 and 11).
8. Since the most resistant rocks have little pore space and the least resistant rocks (unconsolidated sediments) have much pore space, it is evident that the least resistant ones can be made resistant only by the process of consolidation (Fig. 7 and Tables 10, 11, 12, and 14).
9. The two outstanding characteristics of the data in Table 14 are the constancy of saturation (Col. 5) at the time of compaction and the lack of uniformity of density (Cols. 1 and 2) upon completion of the compaction effort.
10. Obviously, sediments of the highest density and least capacity for moisture are the most resistant to traffic and water. It is also obvious that those having the least density and greatest capacity for water are the least resistant (Table 14). By increasing the compactive effort to a magnitude that

would reduce optimum moisture to a nearly constant value, the density and moisture capacity in most cases, if not all cases, could have been reduced to a nearly constant value. However, micaceous and one-size sediments would continue to have large capacity for moisture.

11. To render sediments—especially clays—the most resistant to swell, they must be made very compact with pore spaces from 80% to 90% saturated (Tables 14 and 15).

12. To prevent sediments, especially clays, from losing moisture by evaporation, they must be surfaced with a covering that prevents undue evaporation.

13. Excessive evaporation losses from clay roadbeds can be prevented by the application of a thin blanket of loose sand, although a rigid surface (sand-clay, sand-asphalt, etc.) is superior.

14. Money spent in adequate roadbed compaction becomes an asset to the future road surface or pavement. Indeed, it may warrant the use of a less thick pavement at a considerable reduction in cost (Table 12).

In conclusion, the writer expresses his deep appreciation for the several discussions submitted.

AMERICAN SOCIETY OF CIVIL ENGINEERS

Founded November 5, 1852

DISCUSSIONS

MOVING LOADS AND RESTRAINED BEAMS AND FRAMES

Discussion

BY CHARLES W. DOHN, CLIFFORD PETERSEN, AND A. A. EREMIN

CHARLES W. DOHN,[46] JUN. AM. SOC. C. E.[46a]—The art of designing statically indeterminate structures, including the determination of maximum moments caused by concentrated loading groups on continuous frame structures, has been advanced materially by the publication of this paper. Although a new method may simplify the design of a great number of statically indeterminate structures, there is always some resistance to its adoption, particularly if it is not widely recorded, discussed, and understood. Design agencies that could use such a method are limited by their respective authorities, usually governmental, to widely published and accessible methods which have been accepted and applied, more or less, as office standards. In one instance, known to the writer—which involved bridge cranes operating on rigid-frame bents with haunched beams, and with a trussed jib that had a variable-section upper chord—the Brumfield method of transmission coefficients[4] was rejected in favor of the Cross method of successive approximations (moment distribution)[2] because: (1) Time would not permit teaching, organizing, and instituting a new method; and (2) the clients wanted the design made by a widely known method that could be accepted quickly, and could be checked and approved in successive steps by a number of other engineers, at different times and places.

When many designs and detailed investigations of the statically indeterminate type must be made for a great many different loadings and loading conditions, the method of transmission coefficients holds some advantages over

NOTE.—This paper by R. C. Brumfield was published in May, 1945, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: October, 1945, by R. W. Stewart, Nathan D. Brodkin, and T. F. Hickerson; November, 1945, by Max W. Strauss, D. A. Mackenzie, and Charles E. Schaffner; December, 1945, by J. Kalb, G. Rolsma and F. W. Gieseking, Stanley S. Schure, and Alexander Dodge; January, 1946, by Frederick S. Merritt, Alfred B. Heiser, and T. B. Rights; and February, 1946, by Phil M. Ferguson, Leon Beskin, and Bruce Jameyson.

[46] With Basic Design Dept., George G. Sharp, Naval Architect and Cons. Engr., New York, N. Y.

[46a] Received by the Secretary December 5, 1945.

[4] "Restrained Beams and Rigid Frames," by R. C. Brumfield, 1941 (revision and expansion of "The Solution of Statically Indeterminate Structures"; printed in mimeograph form by Cooper Union, New York, N. Y.).

[2] "Analysis of Continuous Frames by Distributing Fixed-End Moments," by Hardy Cross, *Transactions*, Am. Soc. C. E., Vol. 96 (1932), p. 1.

those of conventional moment distribution. The busy design office handling a multitude of such investigations cannot afford to ignore its advantages. On the other hand, a design agent or office, which handles only a limited number of statically indeterminate investigations and designs, may well prefer the conventional moment-distribution method.

Computations can be expedited using transmission coefficients in conjunction with computing devices to determine frame characteristics, and to distribute moments, according to the procedure developed by Professor Brumfield. His method is well-adapted to machine computation, and the mechanics of calculation could thus easily be done by a group of computers working under the supervision of a design engineer. The speed of applying the method to any particular design could be increased greatly merely by employing more computers and machines. Furthermore, the method tends to relieve the design engineer of the arduous tasks involved in changes in design and loadings; and it allows him more time and inducement to change the design to a more efficient one, whatever the purpose. The value of the method in redesigns and investigations will also be apparent.

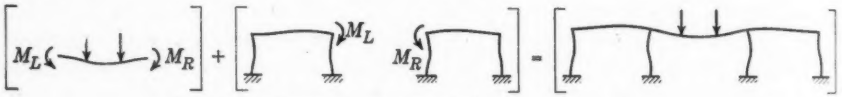

FIG. 30.—OUTLINE OF THE BRUMFIELD METHOD OF ANALYSIS

In the method of transmission coefficients, the only artificially restrained joints are those at the ends of the loaded span and these are only partly locked or restrained. At the beginning (see Fig. 30) the condition can best be pictured as that of a loaded span removed from the structure, with artificial restraint applied at each end of an amount equal to that which would have existed had the loaded span not been removed from the structure. Equal and opposite artificial restraints are then applied to the structure at joints corresponding to the application of the artificial restraints at the ends of the loaded span. The loaded span is then inserted in the structure and the analysis is complete. The method is "exact" and will give the same result as other accepted methods.

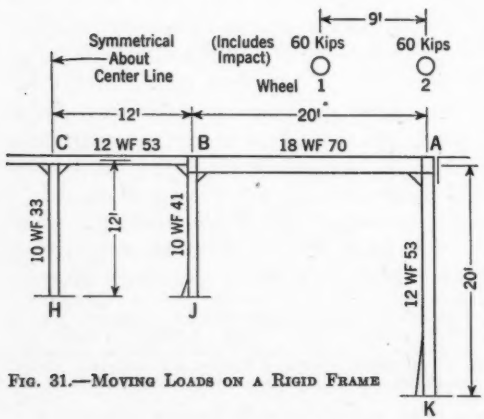

FIG. 31.—MOVING LOADS ON A RIGID FRAME

Moving Loads on a Rigid Frame.—To support a fair appraisal of the method of transmission coefficients, the writer presents, herewith, the solution of a frame with moving loads (Fig. 31). The problem is treated by the Brumfield method and, for comparison, by the Cross method of moment distribution.

By Moment Distribution.—The formal distribution of unit, fixed-end moments at joints A and B, Fig. 31, is presented in Table 11. By a similar procedure, unit ordinates are computed (for the construction of influence lines)

TABLE 11.—ANALYSIS OF MEMBER AB, FIG. 31, BY MOMENT DISTRIBUTION; LOADED AT THE ENDS

Description	(a) End A Rotated					(b) End B Rotated				
	Joint C		Joint B		Joint A	Joint C		Joint B		Joint A
Factor $K = I/L$	0.415	0.415	0.317	0.508	0.730	0.415	0.415	0.317	0.508	0.730
Fixed-end moment	....	....	....	....	−1,000	....	....	....	+1,000	....
Distribution:										
Balance	....	....	....	....	+730	....	....	−317	−508	....
Carry-over	....	....	....	+365	0	....	−158	0	0	−254
Balance	....	....	−116	−185	0	+66	+ 66	0	0	+185
Carry-over	....	−58	0	0	−92	....	0	+33	+92	0
Balance	+24	+24	0	0	+67	....	0	−40	−63	0
Carry-over	0	0	+12	+33	0	....	....	0	0	−31
Balance	0	0	−14	−23	0	....	....	0	0	+22
Carry-over	....	....	0	0	−11	....	....	....	....	....
Balance	....	....	0	0	+8	....	....	....	....	....
Carry-over	....	....	....	+190	−298	....	....	....	+521	−78
Final unit moment	....	....	....	−190	−298	....	....	....	−521	−78

of 1-kip loads placed successively at each fifth-point beginning at end A. Table 12 contains a summary of the results, for negative moment, as applied to beam AB, Fig. 31. The maximum negative moment M_A (see Fig. 32(*a*))

TABLE 12.—UNIT MOVING LOADS ON MEMBER AB, FIG. 31; SUMMARY OF COMBINED UNIT MOMENTS

Description	1-Kip Load at the Following Points (from End A):									
	0.8 L		0.6 L		0.5 L		0.4 L		0.2 L	
	M_B	M_A	M_B	M_A	M_B	M_A	M_B	M_A	M_B	M_A
Fixed-end moment, unit moving load	2.56	0.64	2.87	1.92	2.50	2.50	1.92	2.87	0.64	2.56
Distributed unit load combined with:										
Rotation effect, end A[a]	0.12	0.19	0.36	0.57	0.47	0.75	0.55	0.86	0.49	0.76
Rotation effect, end B[a]	1.34	0.20	1.50	0.22	1.31	0.19	1.00	0.15	0.33	0.05
Combined unit moment, moving load and end rotation	1.46	0.39	1.86	0.79	1.78	0.94	1.55	1.01	0.82	0.81

[a] From Table 11, 1 kip at end A yields $M_A = -0.298$ and $M_B = -0.190$; and 1 kip at end B yields $M_A = -0.078$ and $M_B = -0.521$.

equals $-0.81 \times 60 - 0.70 \times 60 = -90.6$ ft-kips. Similarly (see Fig. 32(*b*)), the maximum negative moment M_B equals $-1.46 \times 60 - 1.40 \times 60 = -171.6$ ft-kips. The final step is the preparation of influence lines for positive moment, from data in Tables 11 and 12. For successive positions of wheel 1 the positive moments (see Fig. 32(*c*)) are:

Fifth-point	Computation	Ft-kip
0.2 L	$\frac{1}{20} \times 60 \times 16 \times 4 - \frac{4}{20} \times 0.82 \times 60 - \frac{16}{20} \times 0.81 \times 60$	143
0.4 L	$\frac{1}{20} \times 60 \times 12 \times 8 - \frac{8}{20} \times 1.55 \times 60 - \frac{12}{20} \times 1.01 \times 60$	215
0.6 L	$\frac{1}{20} \times 60 \times 12 \times 8 + \frac{8}{17} \times \frac{1}{20} \times 60 \times 17 \times 3 - \frac{12}{20} \times 60\,(0.62 + 1.86) - \frac{8}{20} \times 60\,(0.66 + 0.79)$	236
0.8 L	$\frac{1}{20} \times 60 \times 4 \times 16 + \frac{4}{13} \times \frac{1}{20} \times 60 \times 13 \times 7 - \frac{16}{20} \times 60\,(1.46 + 1.40) - \frac{4}{20} \times 60\,(0.39 + 1.02)$	122

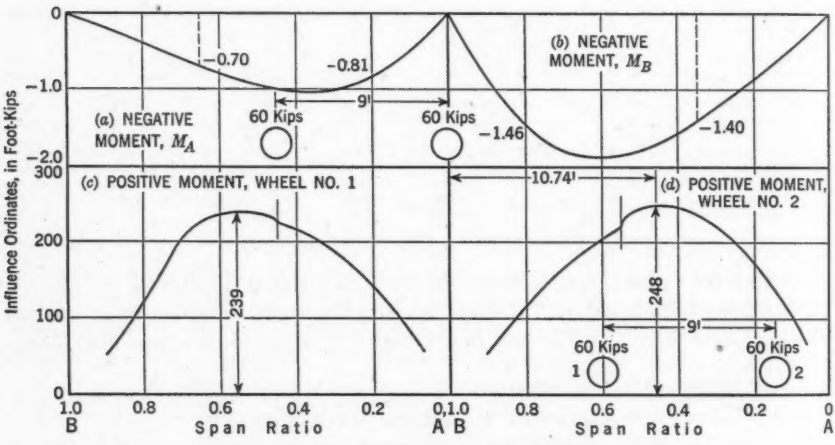

FIG. 32.—INFLUENCE LINES FOR MAXIMUM MOMENTS

The maximum ordinate (Fig. 32(*c*)) is 239 ft-kips. Similarly, for successive positions of wheel 2, the positive moments are:

Fifth-point	Computation	Ft-kips
0.2 L	$\frac{1}{20} \times 60 \times 16 \times 4 + \frac{4}{13} \times \frac{1}{20} \times 60 \times 7 \times 13 - \frac{4}{20} \times 60\,(0.82 + 1.83) - \frac{16}{20} \times 60\,(0.81 + 0.70)$	172
0.4 L	$\frac{1}{20} \times 60 \times 12 \times 8 + \frac{8}{17} \times \frac{1}{20} \times 60 \times 3 \times 17 - \frac{8}{20} \times 60\,(1.55 + 1.20) - \frac{12}{20} \times 60\,(1.02 + 0.28)$	247
0.6 L	$\frac{1}{20} \times 60 \times 8 \times 12 - \frac{12}{20} \times 1.86 \times 60 - \frac{8}{20} \times 0.79 \times 60$	202
0.8 L	$\frac{1}{20} \times 60 \times 4 \times 16 - \frac{16}{20} \times 1.46 \times 60 - \frac{4}{20} \times 0.39 \times 60$	117

The maximum ordinate in this case (Fig. 32(*d*)) is 248 ft-kips, when load 2 is at a distance of 10.74 ft from joint B.

By Transmission Coefficients.—The computation of frame characteristics is presented in Table 13. The next step is to compute the maximum negative

TABLE 13.—BRUMFIELD METHOD OF COMPUTING FRAME CHARACTERISTICS

$$Q = \frac{L}{I}; \quad R = \frac{i^2}{1 - i^2/i^2}; \quad C = \frac{1}{2 + R/Q}; \quad m = \frac{R}{i}; \quad t = \frac{1}{1 + i/R}$$

Member	Stiffness Index		Restraint index R	Transmission Coefficient C		Moment split m			Torque split t
	$Q(2-C)$ =	i		Value	Member				
Joint A:									
AK	0.562 (2 − 0.5)	0.843	0.361	0.378	KA	KA	1.0	..	0.30
AB	0.208 (2 − 0.268)	0.361	0.843	0.165	BA	1.0	BA	..	0.70
Joint B:									
BA	0.208 (2 − 0.165)	0.384	0.358	0.268	AB	AB	0.593	0.72	0.482
BJ	0.650 (2 − 0.5)	0.975	0.228	0.435	JB	0.367	JB	0.28	0.190
BC	0.337 (2 − 0.319)	0.568	0.275	0.355	CB	0.633	0.407	CB	0.328
Joint C:									
CB	0.337 (2 − 0.355)	0.556	0.382	0.319	BC	BC	0.50	0.69	..
CH	0.842 (2 − 0.5)	1.263	0.273	0.430	HC	0.31	HC	0.31	..
CD = CB	0.337 (2 − 0.355)	0.556	0.382	0.319	DC = BC	0.69	0.50	DC	..

moments at the supports in span AB (Fig. 33) caused by the moving loads. By Eq. 28, $A_R = \frac{0.165}{1 - 0.165 \times 0.268} = 0.173$; and, similarly, $A_L = \frac{0.268}{0.956} = 0.282$. The span distribution ratio $\lambda = \frac{r}{L} = \frac{4.5}{20} = 0.225$. The remainder of the solution yields:

Quantity	Left	Right
C	0.165	0.268
K	0.455	0.555
B	0.29	0.29
D	0.30	0.28
M	−170	−90

—in which: K, for maximum moments M_L and M_R, is selected from Fig. 6; B and D are selected from Fig. 5; and M_L and M_R, respectively (in foot-kips), are computed from Eqs. 27. The dissymmetry span ratio ϵ is zero in this case because the modulus of dissymmetry of the particular loading group used is zero.

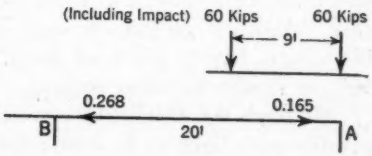

FIG. 33

The foregoing values compare favorably with those obtained by the Cross method (see Figs. 32(*a*) and 32(*b*)).

The maximum negative moments having been computed, the maximum positive moment under the moving loads in the span AB is computed as follows:

By Eq. 32, $q_1 = 0.165 + 2 \times 0.165 \times 0.268 + 0.268 = 0.522$; and $q_2 = 0.268 + 0.165 \times 0.268 = 0.312$. Eqs. 33 then yield: $f_3 = 4 \times 0.522 = 2.088$; $f_2 = 3\left(\frac{4.5 \times 0.522}{20} - 4 \times 0.312\right) = -3.339$; and f (Eq. 33*c*) $= 6 \times 0.522 \times 0.225^2 + 0.312\left(6 - \frac{6 \times 4.5}{20}\right) + 2(3 \times 0.268 - 0.165 \times 1) = -0.112$.

The application of the foregoing equations, as shown, gives the value of κ for use in placing the loads to produce maximum positive moment. Substituting in Eq. 34: $\bar{\kappa} = 2 \times 0.268 - 1 + 0.225(1 + 0.165) + 0.312(6 \times 0.225^2 - 2 \times 0.225) - 0.522 \times 3 \times 0.225 \times 0.225^2 = -0.339$. Finally, by Eq. 35, $\kappa = 2.088 \times -0.339^3 - 3.339 \times -0.339^2 - 0.112 \times 0.339 = -0.312$. This represents the ratio of the distance of the mass center of the loading group from the left support. Hence, wheel 2 is $0.312 \times 20 + 4.5 = 10.74$ ft from joint B for maximum positive moment. As in the determination of maximum negative moment the remainder of the solution, with $\lambda = 0.225$, yields:

Quantity	Left	Right
C	0.165	0.268
κ	0.312	0.312
B	0.24	0.24
D	0.25	0.25
M	−142	−70

in which, as before, B and D are selected from Fig. 5, and M_L and M_R, respectively (in foot-kips), are computed from Eqs. 27. The composite maximum moment is computed by applying the laws of statics, as: $\frac{60 \times 10.74 \times 9.26}{20} + \frac{60 \times 1.74 \times 18.26}{20} \times \frac{9.26}{18.26} - \frac{142 \times 9.26}{20} - \frac{70 \times 10.74}{20} = 246$ ft-kips. This value compares favorably with $M = 248$ ft-kips as computed by the Cross method (see Figs. 32(*c*) and 32(*d*)).

Summary of Comparisons.—The actual design of the bent illustrated in Fig. 31 was better proportioned to resist the moments with minimum excess of material, which was an important requirement of the design. Hence, haunched beams and columns were proportioned to resist the moments as indicated by the preliminary investigation, and another investigation was made. Sidesway was neglected in the preliminary design since the column sections were large and there were enough of them to keep sidesway effect to a minimum in any case. Another condition of loading—a horizontal thrust—required bracing in the interior bay for the final design. The exterior columns were also increased 50% in section for other reasons, chiefly impact on the column itself.

Although the Brumfield method has a short distribution, it requires comparable work in order to obtain the frame characteristics. The Cross method requires little more than a pencil and something to write on. It can be applied any time and anywhere. The Brumfield method with its formulas, constants, and charts is better adapted to advanced design work in a well-established office.

One great advantage of using the Cross method rather than the Brumfield method in the analysis of moving loads should not be overlooked—the principle of separately distributing an assumed fixed-end moment of, say, 1,000 ft-kips at each end of all loaded spans. It is then only necessary to multiply the distributed moments by the ratio of the actual fixed-end moment to the assumed fixed-end moment. The influence lines can then be plotted; maximum and minimum moments for any loading are obtained therefrom. The influence lines are a useful part of any analysis using live loads, and an American design engineer would not want to analyze any involved structural design problem without their aid.

The method of using influence lines will be preferred as the "picture" can be applied to different loading conditions with ease. The Brumfield method can be used to determine influence lines also, although not with the same ease and convenience as the Cross method; the latter requires only two distributions for any number of loadings and corresponding fixed-end moments in any one span (see the foregoing example of moving loads).

In any actual continuous structure subjected to moving loads, the loads outside the span as well as the loads in the span must be considered for maximum and minimum moments. For most practical problems, influence lines are the best guide to the placing of loads for maximum and minimum moments due to loads both within the span and outside the span. Hence, except as in the special case of the span in question loaded and all other spans unloaded (the dead load remaining, nevertheless), there is no point in determining the maximum moments within a span of a restrained beam, especially when the loading group occupies more than two spans. Most actual maximum moments in design work are due to loadings both inside and outside the span in question. Hence, influence lines must be used, and the Cross method is probably the best adapted to their use.

The Brumfield method is not so complicated as might appear. It is probably a bit difficult to assimilate and apply at first. With constant practice, the operations become automatic, and the time and effort necessary for each step gradually diminish. It might be well to note that the frame characteristics are constants. In standard multiple designs they are similar and hence, once calculated, they can be used over and over again for all different conditions of loadings, for secondary stress investigations, and for sidesway, settlement of supports, temperature stress, degree of fixity, erection, damage, replacement, modification, alteration, and conversion.

The Hardy Cross method is more easily recalled to use after long periods of disuse. It requires quite an extensive review to apply the Brumfield method after a long lapse of time. Furthermore, practically minded designers will question the niceties of this method. It is probably only in the unusual problems, or in the case where detailed investigation is required, and where otherwise it is unnecessary to obtain influence data, that the Brumfield method is most applicable. The method is also helpful as a check or alternative method to the Cross method.

In the study and application of moment-distribution methods, the individual designer will usually evolve tables, curves, and nomographic charts as an aid to computation. Several of these have been published, and their usefulness should not be overlooked. It might be possible to construct a nomograph to read M_L and M_R and a maximum positive moment directly based on Professor Brumfield's equations and constants. One of the factors in this nomograph could be fixed-end moments.

The similarity of some of the factors used in determining the "frame characteristics" to equations of electrical circuits suggests that perhaps it would be possible to set up an electronic computing device for "frame characteristics." Then all one would need to do would be to press some buttons and read the frame characteristics on a dial, enter a nomograph, and read M_L, M_R, positive M_{max}, and make a simple distribution on prepared forms. By another electronic computing device it might also be possible to draw influence lines if necessary.

In the hands of the unenlightened, this method, and other useful tools of engineering, can become "sledges" to drive engineers deeper into the mud of tedium. However, in the hands of enlightened direction, with sufficient care in distributing the labor of computation over a large group of people, working short daily hours under healthful working conditions with adequate compensation, and with sufficient rest periods, something worthwhile can be accomplished without straining humanity.

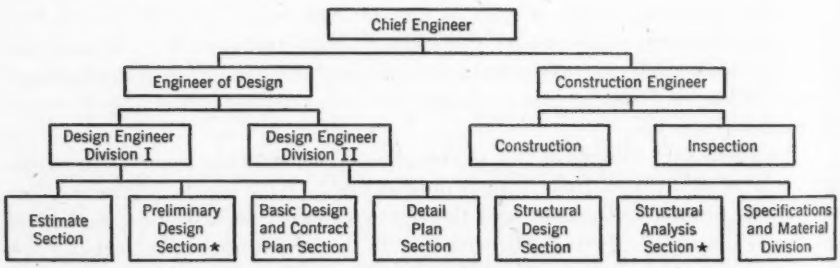

FIG. 34.—ORGANIZATION CHART FOR ENGINEERS ENGAGED IN THE DESIGN AND CONSTRUCTION OF CONTINUOUS FRAMES

Moment distribution is but one step in the design procedure for any continuous structure. To illustrate the proportionately small but nevertheless important part that moment analysis occupies in an engineering organization doing continuous-frame structural work, a simplified organizational chart is presented in Fig. 34. Sections where moment analysis and the Brumfield method may be applied are indicated as the "Preliminary Design" section in division I and the "Structural Analysis" section in division II. The design organization illustrated is one engaged in preparing both contract and working plans, as well as supervising and inspecting construction. In an organization

engaged only in preparing contract plans, moment analysis assumes a proportionately larger part of the organization.

CLIFFORD PETERSEN,[47] JUN. AM. SOC. C. E.[47a]—It would be well to state that Professor Brumfield's method is flexible to the utmost and is readily applicable to the solution of problems in rigid frames outside the scope of conventional rectangular building frames. Also, once a basic understanding of the method has been absorbed, it is possible to pick up the method and employ it on a short notice even if a considerable interval has passed since last it was used.

The writer first became familiar with the method during his undergraduate days. In the succeeding years he has been engaged entirely in the aeronautical engineering field and has had no occasion to apply the method to building frames. There have been a few isolated yet important instances, however, where it has been necessary for the writer to investigate moving air loads and other loads in the design of multiangular frames which occur so often in aircraft structures. After the briefest perusal of Professor Brumfield's mimeographed notes,[3] the method was applied successfully to the exact solution of the problems with a considerable over-all saving in time and effort. The saving in time resulted primarily from the fact that, once the characteristics of the frame were determined, it was a comparatively simple and rapid process to evaluate the effect of the many different loading conditions.

A. A. EREMIN,[48] ASSOC. M. AM. SOC. C. E.[48a]—The distribution moments in continuous frames by the Brumfield method is similar in many respects to the procedure for distributing moments graphically. Therefore, the author's method can be simplified easily by introducing graphical constructions.

For illustration consider a member AB with a constant section and with the ends restrained by the continuity of members. The member AB is subjected to a concentrated load P as shown in Fig. 35. The distances, a and b, to the inflection points in member AB, are

$$a = \frac{C_{AB}}{1 + C_{AB}} \times L \ldots (81a)$$

and

$$b = \frac{C_{BA}}{1 + C_{BA}} \times L \ldots (81b)$$

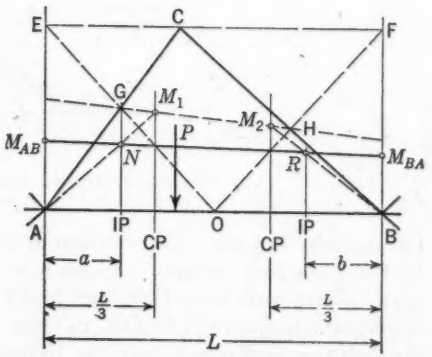

FIG. 35

[47] Aeronautical Research Engr., Univ. of Southern California, Los Angeles, Calif.

[47a] Received by the Secretary February 2, 1946.

[3] "The Solution of Statically Indeterminate Structures," by R. C. Brumfield, 1933 (printed in mimeograph form by The Cooper Union, New York, N. Y.).

[48] Associate Bridge Engr., Bridge Dept., Div. of Highways, State Dept. of Public Works, Sacramento, Calif.

[48a] Received by the Secretary February 13, 1946.

The constants C_{AB} and C_{BA} are determined from Eq. 4. The moments in member AB may be determined by the graphical construction as shown in Fig. 35. Draw the verticals from the inflection points and from the central points at the third-points of span AB. Line EF is drawn through the vertex C of the M-diagram of span AB, as shown. Connect the points E and F with the midspan point O. Draw line GH through the points cut on the M-diagram. Line GH gives the central moments M_1 and M_2 at the third-points. The closing line of the final moments M_{AB} and M_{BA} is drawn through the points cut by lines AM_1 and BM_2.

Fig. 36 illustrates the solution of a continuous beam CAB with a constant section and with the ends restrained by continuity of members. The span AB and the loading are the same as those shown in Fig. 35. At point A the beam is supported on a rigid column. The moments in span CA may be determined either by the author's moment-split factors, Eq. 39, or by the cross lines as shown in Fig. 36(*c*). The location of the inflection points in span CA are de-

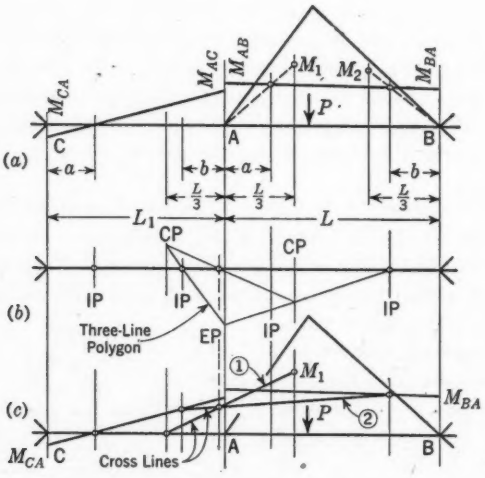

FIG. 36.—GRAPHICAL DISTRIBUTION OF MOMENTS

termined by Eq. 40. The location of the elastic-point vertical is determined [49] by the three-line polygon as shown in Fig. 36(*b*). The central moments in span CA are each zero; therefore, line 1 of the cross lines is drawn through the third-point in span CA and M_1 in span AB. Line 2 of the cross lines is drawn through the closing point on the inflection-point vertical in span AB and the point cut by line 1 on the elastic-point vertical. The intersection point of line 2 and the inflection-point vertical in span CA give the closing point at that span, thus deriving the points by which the closing line of moments M_{CA} and M_{AC} are constructed.

[49] "Analysis of Continuous Frames by Graphical Distribution of Moments," by A. A. Eremin, Sacramento, Calif., 1943 Ed.

In calculating the maximum moment Professor Brumfield has placed the moving load on one span only. It is known that the maximum moment near a support is produced by a moving load placed on two or more spans; and it would be valuable to have Professor Brumfield demonstrate his method for this type of loading. Evidently the method of influence lines is still the simplest for computing such moments. The graphical procedure illustrated by Figs. 35 and 36, in conjunction with Eq. 4 of the paper, is a time-saving method of drawing influence lines. Therefore, it is gratifying that Professor Brumfield has developed the algebraic formulas for the physical properties of structures which may also be applied in the graphical distribution of moments.

AMERICAN SOCIETY OF CIVIL ENGINEERS

Founded November 5, 1852

DISCUSSIONS

TECHNICAL SURVEY—BROOKLYN BRIDGE AFTER SIXTY YEARS

A SYMPOSIUM

Discussion

BY ISIDORE DELSON, AND T. KENNARD THOMSON

ISIDORE DELSON,[22] M. ASCE.[22a]—In presenting the several papers on the Brooklyn Bridge, the authors have done a fine service to the engineering profession.

Mr. Ammann's authoritative paper leaves little for discussion to one who, like the writer, has had an intimate part in the investigation of the bridge.

The writer's preoccupation with the Brooklyn Bridge dates back to 1939, when he was required to make a study of a memorandum on the reconstruction of the bridge submitted by the late Leon S. Moisseiff, M. ASCE.

As far back as 1903, Mr. Moisseiff had collaborated in a plan for reconstruction advocated by the late Gustav Lindenthal, Hon. M. ASCE, then commissioner of the old Department of Bridges of the City of New York. Ever since then Mr. Moisseiff had nurtured the hope of some day participating in such a reconstruction. In his memorandum Mr. Moisseiff stated that:

> "Thorough physical examination and surveys of the material and the structure must be made. An examination of the state of preservation of the existing cables should be made and a substantial number of wires from the cables should be tested for strength, behavior and duration."

The investigation which began four years later led to the recommendation, as stated by Mr. Ammann, that, with motor trucks excluded from the roadways, the two so-called intermediate trusses may be removed, thus providing 30-ft roadways in place of the present narrow roadways which are less than 17 ft wide. Under this plan, as noted by Mr. Ammann, the shallow outer trusses will be rebuilt to the full height of the inner trusses.

As indicated in Fig. 5 (Mr. Seely's paper), four of the six stiffening trusses are directly suspended from the cables. The two intermediate trusses are sup-

NOTE.—This Symposium was published in January, 1946, *Proceedings*.

[22] Cons. Engr., Staten Island, N. Y.

[22a] Received by the Secretary February 21, 1946.

ported on the floor beams about 14.5 ft from the point of suspension of the inner trusses.

The interaction between the floor beams and the intermediate trusses is extremely indeterminate. Entirely neglecting any support given to the floor beams by these trusses, it was found that under a local loading of a 35-ton trolley car, and a 10-ton truck, the maximum stresses in any floor beam were moderate, especially in view of the high strength of the steel.

Inasmuch as the proposed removal of the intermediate trusses is predicated on the carrying capacity of the floor beams, it may not be amiss to discuss the fractures to which Mr. Ammann has referred.

Virtually all these fractures occurred in the top chords of the floor beams close to the point of attachment of the inner suspenders. It was rather baffling to find that all but one of thirty-two fractures recorded between 1929 and 1936 occurred in the side spans, nearer to the anchorages.

A reasonable explanation was suggested to the writer by the very competent bridge foreman at the time the fractures had occurred; namely, that they could have been induced by the braking of the trains approaching the terminals on the downgrades.

Mr. Ammann's paper states the simplifying assumptions made for the analysis of the stiffening trusses. As regards the effect of the diagonal stays, actually each group of twenty-five stays was divided into three zones, in each of which the upward vertical pulls of the stays (the vertical components of the stay tensions) were assumed to be uniform, their resultant acting at the center of the zone. The stays may be conceived as furnishing points of yielding supports to the stiffening trusses.

As regards the very high unit stresses of the bottom chords found in parts of the span in the region of the stays, they were due, to a large extent, to the longitudinal components of the stay tensions, the so-called direct stresses. The original designers had made the rather naive assumption that these direct stresses would be resisted equally by all the longitudinal material on the bridge, both steel and timber. Mr. Gray's paper describes the method used for measuring, in one of the most highly stressed chords, the direct stress due the dead-load stay tension, upon the basis of which participation by the floor system was fixed at 25%. For the direct stresses due to live load, the same ratio was used, in addition to which, participation by the top chords through the web system was determined by analysis.

The revelation of these high unit stresses, in some members exceeding their yield point, was somewhat startling. The figures, of course, were the result of laborious computations based on a theoretical analysis which was set up by the aid of reasonable assumptions for simplifying the complicated structure into a calculable pattern.

Mr. Gray has described the deflection tests under predetermined train loads. At the time that these tests were made on the Brooklyn side span, Mr. Gray also took strain-gage readings on a top chord, which disclosed a stress, 75% of that computed for the particular loading. This compared closely with the result of the computation for stress reduction due to the de-

flection at that point. It was reassuring to find that measure of agreement between the calculated and the measured stress.

T. KENNARD THOMSON,[23] M. ASCE.[23a]—Under the heading, "The Main Cables," Mr. Gray states that "passing trains produced a deflection at the center of the main span in excess of 1 ft." In other words, the vertical deflection from live loads is given as about 1 ft, but of course the vertical variation caused by the expansion and contraction of the cable is much greater. For the Brooklyn Bridge the total possible deflection is 18 ft, 9 ft above and 9 ft below the mean level. For the Williamsburg Bridge the total possible variation is about 6 ft 9 in.

The writer obtained these values from the Bridge Department of New York City, N. Y., for an article that appeared in 1909.[24]

Corrections for *Transactions:* In January, 1946, *Proceedings*, Fig. 6 on page 20, remove one zero from the ordinates.

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[23] Cons. Engr., New York, N. Y.

[23a] Received by the Secretary January 22, 1946.

[24] "New York City Bridges," by T. K. Thomson, *The Engineering Magazine*, September and October, 1909.

AMERICAN SOCIETY OF CIVIL ENGINEERS

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DISCUSSIONS

DESIGN DEVELOPMENTS—STRUCTURES OF THE TENNESSEE VALLEY AUTHORITY A SYMPOSIUM

Discussion

BY JOSEPH R. BOWMAN, PAUL GISIGER, OREN REED, AND R. L. SMART

JOSEPH R. BOWMAN,[18] JUN. AM. SOC. C. E.[18a]—The title of the second paper of the Symposium is rather inclusive, whereas the paper itself is primarily concerned with but one of the several types of hydroelectric power developments designed by Tennessee Valley Authority (TVA). Messrs. Palo and Marks are to be commended, nevertheless, for their emphasis on the importance of functional layout to design. The relative importance of the functions of water flow, flow of electricity, and mechanical control is generally understood and accepted by those engaged in the design of hydroelectric power stations. On the other hand, considerable difficulty is experienced frequently through failure to understand the proper correlation of the electrical control, auxiliary electrical equipment, and auxiliary mechanical equipment functions. The most desirable layout of unit and station auxiliary services can best be obtained through appropriate evaluation of the interrelation of these functions. Evidence of the value of competent functional planning is reflected in the minimum of interference of unit and station service functions presented in many of the TVA hydro plants.

To simplify otherwise cumbersome sentence construction, the writer will use the term "semi-outdoor" with the implication that it includes the "modified outdoor" station of the type used at Hiwassee Dam, as well as the true "semi-outdoor" type of station adapted to the Cherokee, Douglas, Fort Loudoun, Watts Bar, and Kentucky projects.

Notable among the accomplishments of the TVA program is the departure from the conventional indoor power station housing large units. Messrs. Palo and Marks cite relative economy, operating convenience, and appearance

NOTE.—This Symposium was published in October, 1945, *Proceedings*. Discussion on this Symposium has appeared in *Proceedings*, as follows: December, 1945, by A. T. Larned; January, 1946, by William P. Creager; and March, 1946, by I. L. Tyler, and Philip Sporn.

[18] Asst. Civ. Engr., Erik Floor and Associates, Chicago, Ill.

[18a] Received by the Secretary February 7, 1946.

as the general bases for the selection of the various powerhouse types. In addition, they have advanced the conditions of (1) maximum tailwater above the generator floor and (2) access level above the generator floor as the criteria governing the selection of the semi-outdoor type of station. In the comparison of a number of the TVA powerhouses the writer has observed inconsistency and inadequacy in the application of these criteria. For example, these governing conditions, expected at six of the main-river, low-head developments, led to the selection of semi-outdoor hydro stations at Kentucky, Watts Bar, and Fort Loudoun; on the other hand, indoor stations were selected at the Pickwick Landing, Guntersville, and Chickamauga projects.

The appearance factor frequently exerts considerable influence on the selection of power stations employed in public power programs. Whereas some organizations direct attention to the powerhouse by the accentuation of vertical lines, the TVA has utilized accentuation of horizontal lines to direct attention from the powerhouse to other structures on the project. The effect of the latter policy is further promoted by the adoption of the semi-outdoor station.

Several factors are involved in the determination of the relative economy of the indoor and semi-outdoor stations. A considerable saving may often be obtained through the elimination of expensive superstructure walls. However, where very high tailwater conditions are expected, a weight of material equivalent to that eliminated must be added to the substructure to insure the stability of the structure. The net saving, therefore, will be determined by the unit price differential between superstructure and substructure types of construction. Another factor worthy of consideration is the replacement of the conventional roof structure by the deck-type roof used in semi-outdoor stations. The saving, if any, effected by the foregoing procedure may be offset by the increased cost of a massive outdoor gantry crane. Further economy may be obtained by modification of the crane to embrace the functions of handling intake and draft-tube gates (as adapted to the Kentucky station), thereby eliminating the need for separate cranes to perform those operations. From the viewpoint of operating economy, additional expense may be incurred in moving the massive outdoor crane, and in performing operations ordinarily not characteristic of indoor stations, such as the handling of unit hatch covers.

Floor operation (as distinguished from control room supervision) of as few as two or three large units has apparently become standard practice in many hydro plants of the TVA and other organizations. Efficient floor operation usually demands the location of unit auxiliary equipment in the immediate vicinity of the floor operator's station, which, in turn, is usually near the unit governor controls. Design for stability against extremely high tailwater results in very compact substructures in semi-outdoor stations. Consequently, a critical shortage of space for auxiliary equipment near the unit may result. As the physical size of auxiliary equipment is nearly constant over a considerable range in unit sizes, this shortage of space will become even more critical for smaller units, probably necessitating the location of auxiliary equipment some distance from the floor operation center. The compactness of the substructure of a semi-outdoor station frequently involves the elimination of a

distinct turbine floor and necessitates the location of all governing equipment—actuator, sump, and pressure tank—in a compact group at the floor operation center. This is an important advantage in view of the requirements of efficient floor operation. Further operating economy is obtained by the utilization of the "twin" operation center, in which the auxiliary and governing equipment of two adjacent units is grouped into a common operation center. Indoor stations, except when modified by unusual local conditions, ordinarily utilize a distinct turbine floor. This practice frequently leads to the separation of the actuator from the governor sump and pressure tank, thereby increasing attendance requirements. The use of a separate turbine floor often creates unused or "nonproductive" space.

Protection of personnel from severe climatic conditions is frequently advanced as a basic factor governing the selection of indoor powerhouses. The semi-outdoor station appears to satisfy the normal requirements of personnel protection, and the writer believes that the relatively infrequent use of the powerhouse crane alone will not justify the use of an expensive superstructure building. Inasmuch as several outdoor stations are in operation in the northern part of the United States, it is apparent that this type of station has little respect for adverse weather conditions.

The two criteria advanced by Messrs. Palo and Marks for the selection of semi-outdoor stations are merely approximate indicators, and their application should be limited to preliminary studies. On further analysis, the basic factors of appearance, relative economy, and operating convenience will form the bases for final station selection.

As stated by Messrs. Palo and Marks, the rapid pace set in the design of hydroelectric power developments has slowed sufficiently to permit expansive discussion of this subject. The writer notes, however, that much of the technical literature published by the TVA itself has been of descriptive, rather than analytical, nature. The writer suggests that fewer limitations on the exchange of ideas could contribute substantially to the advancement of knowledge in the engineering profession.

PAUL GISIGER,[19] M. AM. SOC. C. E.[19a]—It is a pleasure to note that the commendable policy of the Tennessee Valley Authority (TVA) engineers to report on design and performance of their structures in Society publications has been continued with these highly interesting and instructive papers.

First Symposium Paper.—The first paper by Mr. Riegel is especially valuable because of the data presented on various kinds of observations and measurements of completed structures. In this respect, it is an excellent complement to the Symposium on Dams which was presented at the meeting of the Society's Joint Committee on Masonry Dams in Chattanooga, Tenn., on April 20 and 21, 1939.

In the first part of his paper Mr. Riegel properly states that, in the choice between different types of concrete structures, the cost of future maintenance

[19] Structural Engr., Pennsylvania Water and Power Co. and Safe Harbor Water and Power Corp., Baltimore, Md.

[19a] Received by the Secretary February 6, 1946.

should be a factor and that established methods of construction may also have an influence on design. The writer knows from personal inspection that the concrete in all TVA structures presents an excellent appearance and that maintenance costs in that respect should be very low. It is only fair to record, however, that, in the Tennessee Valley, climatic wear and tear on the concrete is generally less than, say, in Pennsylvania or New England. There are fewer cycles of freezing and thawing, and the same composition of concrete, which in a more northern climate might show excessive water-line deterioration in a short time, may retain its original appearance where winters are less rigorous.

The treatment of uplift conforms to conservative assumptions. As TVA dams generally are standing on sedimentary rock formations with definite bedding and cleavage planes, the policy of drilling uplift relief holes from 20 ft to 40 ft into the foundation is well justified. It has been established elsewhere[20] that even in quite compact rock formations, without such holes, uplift pressures some distance below the foundation joint may be of greater magnitude than those in the joint itself.

Although measured uplift pressures afford a good corroboration of design assumptions with regard to magnitude, they still do not settle the question as to the fraction of foundation area on which they should be considered active. The TVA assumption in this respect, considering the general character of foundations, grouting, and drainage provisions, is on the conservative side. It is highly desirable, but admittedly difficult, to find a method for removing this particular point from the realm of speculation to that of measurable fact.

Mr. Riegel makes it plain that he does not believe in the practice of grouting transverse joints of gravity dams, and the writer fully agrees. Although the reasons given for grouting the lower parts of the joints of high dams are plausible, these questions may still be in order: (1) Will the desired results be obtained? (2) Do not the departure from design assumptions and change in stress conditions thereby produced result in too high a price for the possible benefits? (3) Could not the same benefits be obtained by using drilled holes for grouting, thereby avoiding the obliteration of the contraction joints?

The results of stress measurements at completed structures under load are highly interesting. There are two explanations offered for the discrepancy between measured and observed stresses near the upstream face of some dams. Might not a third (and very probable) one be that the wetted portion of concrete near the upstream face does not shrink, but rather does expand and that the shrinking of the interior portion puts it under compression? The question is also asked whether in Fig. 7(*b*) the vertical stress curve between point 5 and the extreme toe has any basis other than assumption.

From these observational data, the conclusion is inescapable that much about actual stress conditions in dams under load is still to be learned and that the use of elaborate mathematics for the derivation of stresses is of questionable practical value.

The section on "Earth Dams," in Mr. Riegel's paper, gives some excellent illustrations of the principle that sound engineering consists in making the best use of the most readily available materials. It would be interesting to learn

[20] "Uplift Measurements at Holtwood Dam," by Paul E. Gisiger, *Civil Engineering*, July, 1938, p. 449.

how the different materials and cross sections compare economically. Although the earth dams are somewhat overshadowed by the scope of the other structures of the TVA, they are important enough in number and size to justify the hope for a future paper to treat, in more detail, the topics of planning, construction, and performance.

Second Symposium Paper.—The general features of a modern low-head and medium-head hydroelectric unit have become fairly well standardized, and differences in powerhouse cross sections are generally limited to the following: (*a*) The powerhouse may be of the indoor or outdoor (or semi-outdoor) type; and (*b*) step-up transformers may be alongside the generating equipment, or they may be some distance from the powerhouse. Within these general types there may be other variations of a lesser order of significance, but with considerable influence on detail structural design. Such factors are: (1) Location of thrust bearing (above or below the generators); (2) location of transformers (on the upstream or downstream side of the powerhouse); (3) location of mechanical control equipment (all on the main floor, or part of it on an auxiliary lower floor); and (4) differences in complexity and space requirements of electrical apparatus in accordance with system operating necessities.

The irreducible minimum requirements of a hydro powerhouse may be assumed to be reached with a layout in which transformers are located in a separate switchyard, in which generator leads are taken directly away from the powerhouse without interposition of a low tension bus, in which all speed-control equipment is on the main floor, and in which no superstructure is provided. The Fort Loudoun powerhouse comes very close to this minimum layout. It is significant that earlier powerhouses of the TVA are more complex. In general, it will be easier to get along with a minimum of apparatus if a new development is only a fractional addition to an already existing large generating system. On the other hand, if a new development is a main component or large fraction of a power system, its layout tends to be more complicated and to require more space. Apart from the influence of operational and system requirements, personal judgment and organizational preferences may come into play, too. This may be the case to a considerable extent in the provision or omission of a generator-room superstructure. Although it has been stated sometimes that climate need not be a determining factor in deciding whether or not to provide a superstructure, it still appears significant that, to the writer's knowledge at least, no outdoor or semi-outdoor types of powerhouses are in existence in Canada or New England. In TVA territory real blizzards are not a regular feature of every winter and operational difficulties from the omission of the superstructure should not be serious. It will be a matter of great interest to learn how the operating personnel of the TVA will evaluate the relative merits of indoor and outdoor layouts. Local conditions and other features of the general layout may have a decisive influence on the saving possible from omission of superstructure. Comparison of the Chickamauga cross section with that of Hiwassee makes it obvious that in the former the omission of side walls and roof would have meant a relatively much less saving than in the case of Hiwassee.

It is to be noted that, on almost every TVA main-river development, maximum tailwater elevation is above the top of the generators, requiring massive downstream walls of great height. This necessity, although apparently typical of Tennessee River developments, is by no means general. It is quite apparent that, in case of the Pickwick and Chickamauga, this condition worked against the outdoor type of powerhouse.

A comparison of the Fort Loudoun and Chickamauga cross sections, which are otherwise very similar, suggests the question as to why, for Fort Loudoun, the semi-outdoor powerhouse was preferred.

In the section on "Substructures for Large Propeller Units," Messrs. Palo and Marks rightly stress the importance of transverse joints, the attention that should be given to the subdivision, size, and sequence of concrete pours, and the difficulties that may have to be solved to keep commitments for future units within certain limits. The writer fully agrees that, for unit spacings of 40 ft or more, there should be contraction joints between all units. Messrs. Palo and Marks mention the difficulty presented by the usual offset between center line of unit and center line of intake and the resulting nonsymmetry of the draft tube, if the contraction joints are to be in one plane and perpendicular to the powerhouse axis. If there is an electrical bay on the upstream side of the generator room (which means heavily loaded columns to be located over the ends of the intermediate intake piers, as at Pickwick Landing), there are additional difficulties from this source. The writer does not believe that serious disadvantages would result if, on account of these difficulties, transverse contraction joints should deviate from a straight line or a single plane provided that thereby their proper function (to allow volume changes in the direction of the longitudinal powerhouse axis) was not impaired.

Conflicting requirements of hydraulic streamlining and stress concentrations in the noses of intermediate draft tube as well as intake piers may be satisfied with the help of structural or cast-steel vertical members at these locations.

With regard to the complex matter of scroll-case design, the writer feels that, although the efforts of the designing engineers so far have produced acceptable (but by no means perfect) results, a real mathematical treatment never has been undertaken. Such a task is not too complex and should be accomplished some day. The conception of vertical beams between top and bottom slabs is obviously only an approximation of actual conditions. It follows from any assumption of vertical beam action that such action must be accompanied by circumferential stress; and this stress, rather than temperature, is responsible for vertical cracks in scroll-case walls. As a rule every scroll-case wall has one vertical plane of least cross section on its downstream side, approximately on the transverse center line of the unit. The writer has seen vertical cracks on a number of concrete scroll cases at this location and believes they are caused by circumferential stress from water load.

Mathematical investigation of this condition is in line with the conclusion by Messrs Palo and Marks (with which the writer fully agrees): That the field of powerhouse design is at a stage where the course of development has slowed down sufficiently to make desirable a review of present methods and practices,

and application of refinements which were not feasible in the days when each new development was an advance in size and scope over preceding developments.

Messrs. Palo and Marks are to be congratulated for a capable and concise presentation of a complicated, and never fully described, phase of civil and hydraulic engineering.

Third Symposium Paper.—During the past ten years or fifteen years, the development of water power has attracted much more attention than the construction of steam power plants. Therefore, when an organization which made history in the design and construction of water power developments builds a large steam power plant, its accomplishment in this respect deserves careful study by all engineers interested in either steam or hydroelectric power plants.

The paper by Mr. Meyer, intentionally, does not include any discussion of the reasons why, even in a hydro power system like that of the TVA (which includes both a number of run-of-river, as well as high-head storage plants), the addition of a considerable block of modern steam capacity was desirable. The fact that this steam plant was built, however, indicates that a combination of hydro and steam power is very often desirable and economical and that the two kinds of power are not competitive (as is sometimes believed) but complementary.

Mr. Meyer rightly observes that civil engineering features in a steam power plant are quite important for the over-all economy of the station. It is possible that sometimes in the design of steam power plants these features have not received the attention that they should because they are usually overshadowed by the much greater cost of steam producing and generating equipment.

At first it may be somewhat surprising that the Watts Bar station is not directly on the riverbank; but convincing arguments are presented to show that the location is the most economical one after due consideration of all factors involved.

The general design of the station with the unit layout of all auxiliaries follows generally established practice in steam plant design. In view of the outdoor type of powerhouse used by the TVA at the adjacent Watts Bar hydro plant, as well as at several other hydro stations, it is of interest to note that the usual complete housing of boilers, generators, and associated equipment has been chosen for the steam plant. It is perhaps rather surprising that the superstructure of this station was made of brick, whereas the TVA, so far, has mostly used reinforced concrete for hydro station superstructures. The question may also be asked whether it would not have been preferable to put a construction joint into the superstructure between each of the four units. It does not appear quite logical to have construction joints in the substructure but not in the superstructure, which is more subjected to dimensional changes induced by temperature variations.

The steel-frame design of the generator-room roof follows the precedents established by the hydro plants of the TVA and it is an improvement over the conventional truss layout in several ways. Mr. Meyer rightly calls attention

to the ingenious design of the crane-rail brackets in which the possibilities of welded construction have been utilized fully and intelligently.

With regard to the turbo-generator foundations, the question is in order as to whether the use of reinforced concrete was dictated by wartime considerations of material priorities or whether the TVA engineers considered this type of foundation generally preferable to a structural steel foundation. It appears that, with the multitude of piping around the foundations of steam turbines and the relatively greater ease with which piping can be accommodated through and between steel foundations, and also because of the greater speed of erection, there would be considerable advantages in the use of structural steel. However, if (as Mr. Meyer states) vibrations of the completed structure in service are hardly noticeable, the designers are to be congratulated for the results achieved, considering the number of cases in which excessive vibration has occurred.

With regard to the circulating water system, it appears that the plan for taking water out of the upper pool and returning it to the tailrace of the adjacent hydro plant was the proper method for this location. In view of the situation at another steam plant alongside a hydro plant—where it was found advantageous to take circulating water from the hydro plant forebay, but to pump it back so as not to lose any hydro generation—the question may be asked whether the latter system would have been used if the steam plant could have been located closer to the hydro plant. It may be perfectly justifiable, economically, to take water away from the hydro plant for this purpose; but there is a certain suggestion of wastefulness in having to use an energy dissipator at the point where this water is returned to the river. The question also may be asked as to why two conduits were used, considering that one pipe of 9-ft diameter would have been more than equivalent to the two pipes of 6-ft diameter. Since only one intake has been provided and 99% of all troubles in condensing water systems occur either at the intake or at the pumps, it does not appear that the two conduits could be justified by any consideration of dependability.

A somewhat more extensive description of the coal unloading and handling facilities would have been welcome. Such features have a considerable influence on the operating cost of a steam power plant, but are seldom described or treated in literature.

OREN REED,[21] M. AM. SOC. C. E.[21a]—Powerhouse design of the Tennessee Valley Authority (TVA) is presented in a very clear and comprehensive manner in the second paper of the Symposium. Most of the text and illustrations apply to the main-river plants of the TVA, where the powerhouse is an integral part of the dam. At the higher dams of the tributary storage installations, the powerhouse is at the toe of the dam but is usually separated from the dam by a longitudinal contraction joint.

The typical semi-outdoor powerhouse is an economically sound choice for the usual installation where an extremely high tailwater does not require consideration. The heavy, overhead gantry crane cannot reach anything except

[21] Constr. Engr., TVA, Fontana Dam, N. C.

[21a] Received by the Secretary February 9, 1946.

the generator, which is as disadvantageous for maintenance and repairs under operating conditions as it is initially during construction. It is sometimes necessary during construction to leave a part of the roof off the generator room temporarily to place certain apparatus properly. Such interferences with proper construction procedures are unsatisfactory but must often be countenanced because of late deliveries of materials.

Provisions must be made to take care of many visitors at power projects near large centers of population or on good roads. Where large numbers of visitors are expected, more attention must be paid to the architectural appearance of the powerhouse than might otherwise be justified. Both the interior and the exterior of an indoor-type powerhouse have better appearances than an outdoor or semi-outdoor powerhouse.

Powerhouse design of recent installations by the TVA has been well planned. There is no waste space but there is no evidence of overcrowding. The auxiliaries are conveniently arranged and all reasonable safety precautions have been taken. Facilities for visitors are ample and the general appearance of each installation is very satisfactory.

R. L. SMART,[22] M. AM. SOC. C. E.[22a]—The economic depression of the 1930's and World War II caused the development of hydroelectric projects virtually to stop except for the activity of certain government agencies such as the Tennessee Valley Authority (TVA). Engineers, interested in this field, have followed the developments of the TVA, and thus the paper on hydroelectric stations by Messrs. Palo and Marks is very timely. In a careful review of this paper, the advancement in ideas and methods that took place from the initial work at Pickwick Landing until the final work as represented by Fort Loudoun is described lucidly.

The various functions of a hydro station and their relation to each other are well presented. Far too many projects in the past indicate very clearly that the first function received all the attention and that the remainder became incidental—to be taken care of in almost any way. Simplicity of layout and the elimination of waste areas are of prime importance. Certainly one way to secure such a result would be to fit the installation into the topography of the site, with due consideration for the hydraulic conditions and the most logical access.

If such conditions indicate a distinct advantage for any one type of structure—namely, indoor, outdoor, or semi-outdoor—that type should be chosen. Many engineers have a distinct preference for some one type of structure and have allowed that preference to rule regardless of whether that type was most suitable to the conditions. The TVA most certainly took all conditions into consideration and produced layouts to suit those conditions. All functions of such layouts were properly related for simplicity and the requirements of the project.

The proposal developed by TVA, for the acceptance of uniform waterways by all turbine manufacturers, was a most noteworthy advance and one of par-

[22] Project Mgr., Ambursen Eng. Corp., Cons. Engrs., Houston, Tex.

[22a] Received by the Secretary February 25, 1946.

ticular importance to those engineers who are not in a position to purchase turbines previous either to the initiation of designs, or even to the construction of certain parts of the project itself. It is not at all difficult for turbine manufacturers to agree on a uniform waterway (exclusive of the scroll itself). That one item permits the installation of future draft-tube sections at the time of original construction. By so doing the later need of troublesome, and often very costly, cofferdams can be eliminated, as such provision can be incorporated into the initial construction as part of the ultimate structure.

Modern trends in architecture have produced structures for the TVA that are most in harmony with the entire project. The function of the architect should be emphasized to the fullest degree, not only for outward appearance, but for the development of the proper features within. Although they will not admit it, most engineers are poor architects, and many a powerhouse stands today an eyesore both within and without. Rigid frames, be they steel or concrete, modern welding developments, and streamline designing are willing and able tools for the architect. By virtue of economic conditions prevailing during much of its construction program the TVA was extremely fortunate in being able to assemble a group of engineers with many and diversified lines of thought. They combined to bring forth numerous advancements in design and construction methods that it is hoped will be made available to the engineering world in general. Not all were perfect, not all will be accepted, but there are none not worth knowing.

AMERICAN SOCIETY OF CIVIL ENGINEERS

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DISCUSSIONS

MATHEMATICAL EXAMINATION OF THE HIGHWAY TRANSITION SPIRAL

Discussion

BY WILLIAM T. PRYOR

WILLIAM T. PRYOR,[12] ASSOC. M. AM. SOC. C. E.[12a]—Messrs. Eichler and Eves have presented their paper for the purpose of clarifying certain features regarding the spiral curve. They recognized that it is the aim of highway engineers to construct a highway capable of carrying high-speed traffic with maximum safety and driving comfort. They wrote in Section 1:

> "Of the many problems involved, that of horizontal curvature, accompanied with proper superelevation, is one of the most important, and it is essentially with this feature of the future road that this paper is concerned.

The large Bibliography given by the authors, although not complete, will show that they selected the "clothoid"—the transition spiral—the curve which has a uniform change in length of radius in exact inverse ratio to its length. There are other forms of curvature transition or easement; but, upon careful analysis and by actual use, none have proved to be as easy to handle and as adaptable to highway alinement design and to field staking.

A moving vehicle cannot, safely and comfortably, change immediately from straight-line motion to constant angular motion. Circular curves possess a constant angular change per unit length of arc (the degree of circular curve, D). The transition spiral does not have a constant angular change per unit length, but it does have a uniform rate of angular increase (29)[12b] per unit length (the spiral degree of curvature increase, k). A logical way of using these angular changes to design the spiral curve in conformity with the speed of the vehicle and the superelevation and dependable friction factor on the highway are of vital importance.

NOTE.—This paper by John Oran Eichler and Howard W. Eves was published in May, 1945, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: November, 1945, by Joseph Barnett, Ralph W. Stewart, Donald Thompson, T. F. Hickerson, Allen H. Brownfield, René S. Pulido y Morales, and C. C. Wiley; and December, 1945, by Max C. Koehler.

[12] Highway Engr., Dept. of Design, Public Roads Administration, Federal Works Agency, Washington, D. C.

[12a] Received by the Secretary February 13, 1946.

[12b] Numerals in parentheses, thus: (29), refer to corresponding items in the Bibliography (see Appendix of the paper), and at the end of discussion in this issue.

At the end of the spiral curve the rate of angular change attained produces a computable centrifugal force for any given vehicle speed; and for the same speed the rate of angular change and centrifugal force are halved at a point midway between the beginning and end of the spiral curve. This persists proportionally throughout its entire length. On the circular curve the centrifugal force and angular change are constant for any given vehicle speed.

An easy procedure for applying the basic formulas in the geometric design of highway alinement is demonstrated by the diagrams in this discussion. Fundamentally, this discussion has been prepared to show, in a graphical manner, the general application that can be made of the authors' developments to serve as a guide in choosing spiral curves for transition in highway alinement.

Tables are available (2)(59) which will be found very useful for the solution of most of the functions of spiral curves and circular curves for use in design layout and field staking, thus avoiding the labor of solving long-series equations. The use of tables that have been calculated and found sound in actual practice should be emphasized. Field and office men alike hesitate to begin the use of long formulas, and recoil quickly from such formulas if some other method will yield the same results.

The superelevation, the friction factor (expressed as a cross slope to be in the same units as superelevation), degree of circular curve, and the spiral degree of curvature increase are all intrinsically related. These, as well as the geometry of the spiral curve, must be clearly understood if intelligent use is to be made of all principles. Then full application can be made of the flexible features of the basic formulas.

If the velocity, Eq. 1, is measured in v miles per hour instead of in V feet per second:

$$\frac{P}{W} = \frac{0.0669\, v^2}{r} \qquad (89)$$

The ratio P/W is illustrated in Fig. 11(*a*). The centrifugal force P must be opposed equally or the moving vehicle will slide outward off the curve. Op-

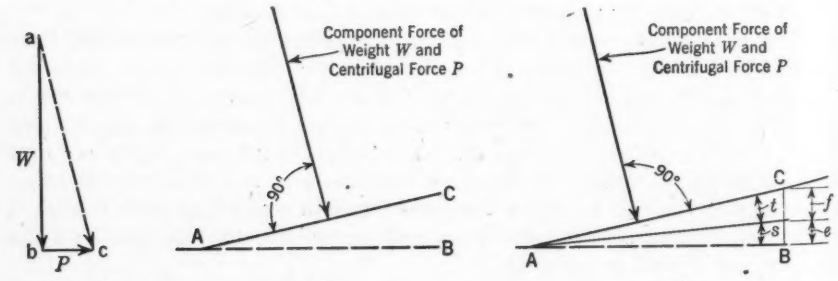

FIG. 11

position may be accomplished by inclining the curving path so that its cross slope is everywhere normal (at right angles) to the component force of weight W and the centrifugal force P, Fig. 11(*b*). Angle CAB may be thought of as

being the sum of two angles, s and t, Fig. 11(*c*). The tangent of the sum of these two angles is e (superelevation) plus f (friction factor). Considering the length of line AB as unity, by similar triangles $\frac{(e+f)}{1} = \frac{P}{W}$, and from Eq. 89 if D, in degrees per one hundred foot arc, is used instead of $1/r$:

$$(e+f) = 0.0000117\, D\, v^2 \ldots\ldots\ldots\ldots\ldots\ldots (90)$$

Use of Eqs. 89 and 90 made it easy to prepare Fig. 12 to show the relationship of degree and radius of circular curve for vehicle speeds, superelevation, and friction factors. The sum of e plus f, Fig. 11(*c*), is shown as the abscissa in Fig. 12.

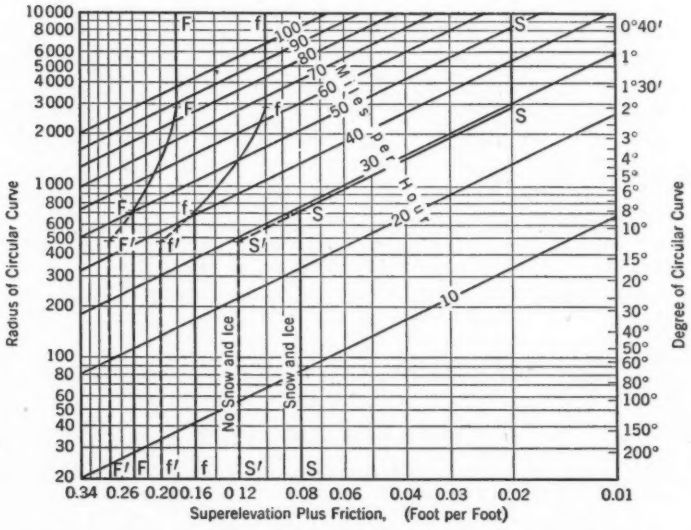

FIG. 12.—DEGREE AND RADIUS OF CIRCULAR CURVE FOR VEHICLE SPEEDS, SUPERELEVATION, AND FRICTION FACTORS

In the use of Fig. 12, line SSSS represents the superelevation rate suggested for circular curves in climatic regions where snow and icy conditions may be expected on the road. It was computed by Eq. 90, setting f equal to zero and v equal to 30 miles per hr. Line SSS′S′ represents the superelevation rate suggested for circular curves in climatic regions where no snow or ice is expected on the roadway. Line SSS′S′ has an upper limit of 0.12 ft per ft for a 12° circular curve, and all sharper circular curves. Use of this upper limit to the superelevation rate is considered the maximum for safety and comfort where road surface conditions are likely to be good throughout the entire year, with no snow or ice on the road at any time. Line ffff represents 0.08 ft per ft of friction added to the line SSSS, and line fff′f′ the same amount to line SSS′S′. Likewise, line FFFF represents 0.16 ft per ft of friction added to line SSSS, and line FFF′F′ the same amount to line SSS′S′.

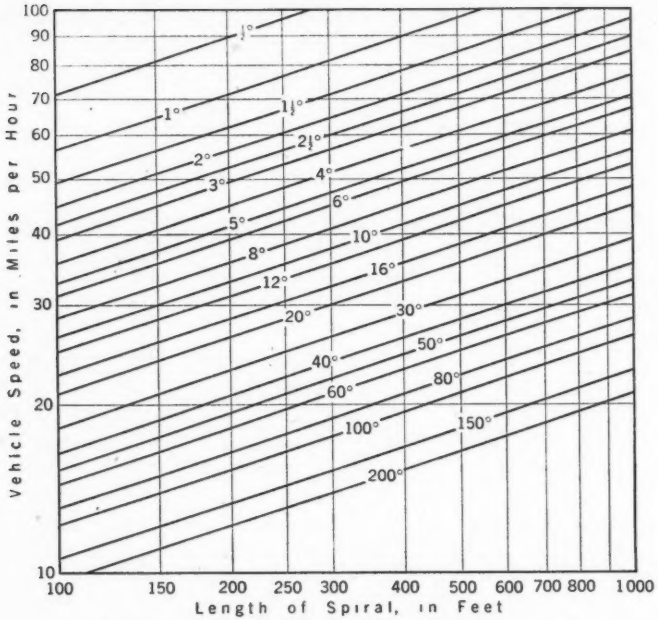

FIG. 13.—LENGTH OF SPIRAL CURVE, ACCORDING TO VEHICLE SPEED AND DEGREE OF CIRCULAR CURVE

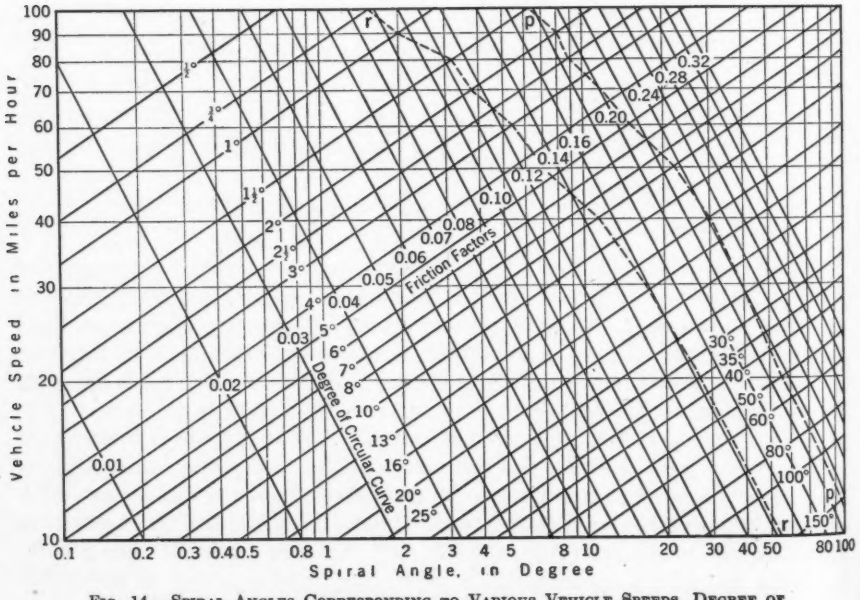

FIG. 14.—SPIRAL ANGLES CORRESPONDING TO VARIOUS VEHICLE SPEEDS, DEGREE OF CURVE, SUPERELEVATION, AND FRICTION FACTORS

A circular curve may be selected easily and quickly for any combination of conditions within the practical limits of e, f, $(e + f)$, and maximum vehicle speeds of from 10 to 100 miles per hr. This represents a graphical method of showing the results obtainable by the full use of Eq. 1 of the paper. For spiral curve design the superelevation-plus-friction factor, the degree of circular curve, vehicle speed, and rate of change of radial acceleration (or the spiral degree of curvature increase) are combined.

Eq. 16*b* can be reduced to more easily used terms. Change from $1/R$ to D in degrees per 100 ft-station of arc and from V in feet per second to v in miles per hour; and:

$$S = \frac{0.0005506\ D\ v^3}{\gamma} = \frac{D\ v^3}{1{,}816\ \gamma} \qquad (91a)$$

and

$$\phi = \frac{0.000002753\ D\ v^3}{\gamma} = \frac{D^2\ v^3}{363{,}200\ \gamma} = 20{,}202\ \frac{(e+f)^2}{\gamma\ v} \qquad (91b)$$

Fig. 13 was constructed simply by substituting appropriate values for the respective variables in Eq. 91*a* and plotting the S-answers. The rate of change of radial acceleration (γ) in feet per second^3 was set at one, as explained sub-

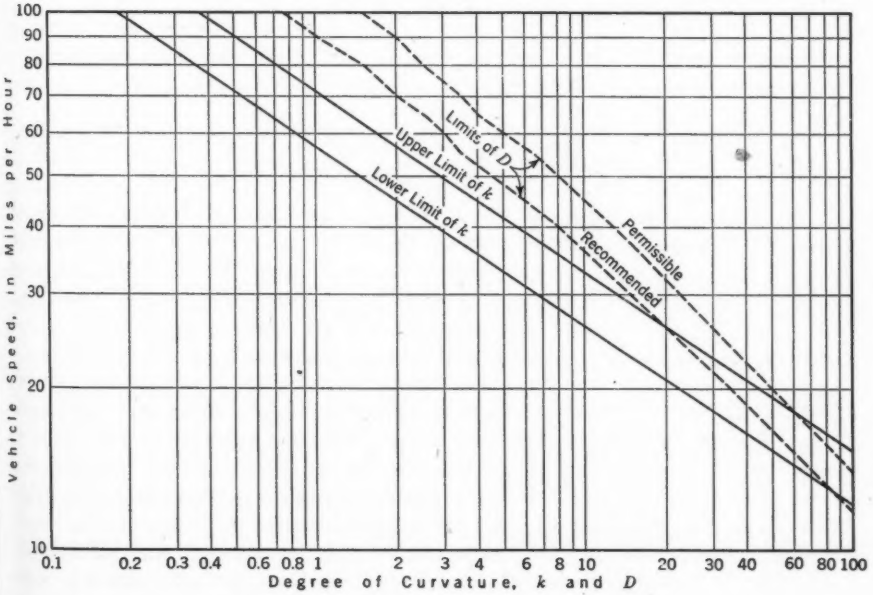

FIG. 15.—DEGREE OF SPIRAL AND CIRCULAR CURVES FOR DIFFERENT VEHICLE SPEEDS

sequently herein. Fig. 14 was constructed by use of Eq. 91*b* with $\gamma = 1$. In design, if a value of $\gamma = 2$ is used, spiral angles will be exactly one half of those shown. Line pp in Fig. 14 indicates the limit of the sharpest permissible circular curves; and the line rr indicates the limit of recommended curves.

Fig. 15 shows the relationship between the upper limits of degree of circular curve D and the spiral degree of curvature increase k for vehicle speeds of from 10 to 100 miles per hr. The limits of D were chosen from critical points in Fig. 12, and the limits of k were calculated by simple manipulation of Eq. 91*a*. Since $k = D/S$, if S is measured in 100-ft stations:

$$k = \frac{181{,}600}{\gamma\, v^3} \qquad (92)$$

For the upper limit of k a value of $\gamma = 2$ was chosen, and for the lower limit, $\gamma = 1$.

In order to ascertain, rationally, the value for γ to be used in Eqs. 91*a* to 92 so that they will satisfy the requirements for safety, and provide comfort and ease in gaining the radial acceleration for the driver while the vehicle is moving around the spiral curve, four items of fact should be kept in mind, as follows:

(1) Vehicles will be driven over the highway at speeds of from 10 to 20 miles per hr faster than the speed rating of the highway, especially for passing;

(2) As a factor of safety, designers should plan for this faster moving vehicle when and wherever it is possible, and then the slower moving vehicles will be provided for;

(3) Steering wheels must be turned much more when the vehicle is being driven considerably faster than the speed for which the curve is superelevated, in order to generate sufficient friction to counteract the unbalanced centrifugal force; and

(4) Although R. A. Moyer, Assoc. M. Am. Soc. C. E. (19) indicates that γ should be equal to 2, W. H. Shortt (27) uses a value of $\gamma = 1$.

By casual examination of the V^2/r-values in Table 3 it is evident that, for $\gamma = 2$ ft per sec^3, the time to attain these rates of radial acceleration would be from 1 sec to 5 sec. Actually the circular curves and vehicle speeds giving V^2/r-values greater than 9 ft per sec^2 require $(e + f)$-values that are too high, relatively, for safety and comfort and ease in traveling around the curves. Therefore, if γ is fixed at 2 ft per sec^3, all acceleration rates would be attained in less than 5 sec. This is not much time, but perhaps the efficient and alert vehicle driver will not experience any difficulty and will experience not too much discomfort in the process. In the light of the foregoing analysis, most engineers will undoubtedly agree that $\gamma = 2$ should be the largest value used in design.

TABLE 3.—SUPERELEVATION PLUS FRICTION $(e + f)$ FOR GIVEN RATES OF RADIAL ACCELERATION

V^2/r (ft per sec^2)	$(e+f)$ (ft per ft)
2.0	0.06
3.0	0.09
4.0	0.12
5.0	0.16
6.0	0.19
7.0	0.22
8.0	0.25
9.0	0.28
10.0	0.31

It is believed, however, that a smaller value of γ (say, $\gamma = 1$) would be preferable. In Table 4 the vehicle speed in miles per hour (Col. 1), the degree of circular curve (Col. 2), and recommended length of spiral curve (Col. 3) were taken from Figs. 12 and 13, respectively, and other columns were computed

for: The vehicle speed rating of the highway (Cols. 4 and 5); the rated speed plus 10 miles per hr (Cols. 6, 7, and 8), and the rated speed plus 20 miles per hr (Cols. 9, 10, and 11).

TABLE 4.—COMPARATIVE EFFECT OF EXCEEDING SAFE DESIGN SPEEDS ON A HIGHWAY

Design speed (miles per hr)	Maximum curve[a] (degrees)	Spiral length[b] (ft)	At Design Speed[c]		10-Mile Excess[d]			20-Mile Excess[e]		
			V^2/r	$(e+f)$	V^2/r	$(e+f)$	γ	V^2/r	$(e+f)$	γ
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)
100	1.5	820	5.6	0.175	6.8	0.212	1.3	8.1	0.252	1.7
90	2	800	6.1	0.189	7.5	0.234	1.4	9.1	0.283	1.8
80	2.5	700	6.0	0.187	7.6	0.236	1.4	9.4	0.292	2.0
70	3.5	660	6.4	0.200	8.4	0.261	1.5	10.6	0.331	2.1
65	4	600	6.3	0.197	8.4	0.262	1.5	10.9	0.337	2.3
60	5	600	6.8	0.210	9.2	0.286	1.6	12.0	0.373	2.3
55	6.5	600	7.4	0.229	10.3	0.320	1.6	13.7	0.426	2.5
50	8	550	7.5	0.234	10.8	0.336	1.7	14.7	0.458	2.7
45	10	500	7.6	0.237	11.4	0.353	1.8	15.9	0.493	3.0
40	13	460	7.8	0.243	12.2	0.379	1.9	17.6	0.546	3.4
30	23	340	7.8	0.242	13.8	0.430	2.4	21.6	0.671	4.7
20	50	220	7.5	0.234	16.9	0.525	3.4	30.1	0.934	8.0
10	200	110	7.5	0.234	30.0	0.935	8.0	67.5	2.10	27.0

[a] The sharpest permissible curve. [b] Recommended length of spiral curve. [c] For the speeds given in Col. 1, $\gamma = 1$ in each case. [d] Results for 10 miles per hr in excess of speeds shown in Col. 1. [e] Results for 20 miles per hr in excess of speeds shown in Col. 1.

It is quite obvious that the vehicle driver who attempts to exceed the safe speed of the highway, either at constant speed or by increasing his speed, to pass another vehicle quickly creates $(e + f)$-conditions that are not safe and comfortable, but hazardous. This is especially so for vehicles moving at speeds exceeding the vehicle speed rating of a slow-speed highway. Fixing a value of $\gamma = 1$, however, keeps the value of γ in Col. 8 (Table 4) between limits of 1.3 at 110 miles per hr and 3.4 at 30 miles per hr, and, in Col. 11, between the limits of 1.7 at 120 miles per hr and 4.7 at 50 miles per hr. These resulting values for γ are then near or within the maximum value of $\gamma = 2$ established previously, except in those cases where the highway speed ratings of 10 miles per hr and 20 miles per hr (Col. 1, Table 4) are exceeded by 10 miles per hr (Cols. 6, 7, and 8) and those of from 10 to 40 miles per hr (Col. 1), are exceeded by 20 miles per hr (Cols. 9, 10, and 11). In view of this analysis, the graphs, Figs. 13, 14, and 15, were prepared by making $\gamma = 1$ ft per sec^3, which should be the desirable upper limit.

Fig. 14 is a graph designed to give the total spiral angle, in degrees, for vehicle speeds of from 10 to 100 miles per hr, the degree of circular curve, and superelevation-plus-friction factor, as well as the sharpest circular curve permissible and recommended. Because of these features, it is a composite graph which may be used in lieu of Figs. 12 and 13. For example, if the vehicle speed rating of the highway is to be 60 miles per hr and a spiraled 3° 00′ circular curve will fit the topography, the spiral angle should be 5.4°, and only 0.126 ft per ft of superelevation plus friction would be required to balance the centrifugal force. Moreover, for example, let us assume that the central angle

($\triangle$) between two tangents is 20° 00′, and that it is desired that the curve joining these straights be transitional throughout, and the vehicle speed rating of the highway is to be 60 miles per hr. (It is not a recommended practice to design curves transitional throughout, but it is useful as a means of demonstrating the easy application of a flexible procedure.) Proceed from the line marking the spiral angle of 10° $\left(\frac{20°}{2}\right)$ on the graph and follow the line upward until it intersects the vehicle speed line of 60 miles per hr. These two lines intersect at almost the same point as the 4° circular curve and 0.17 ft per ft superelevation-plus-friction factor line. Use of a superelevation rate of 0.04 ft per ft would mean 0.13 ft per ft of friction would be required. Almost at a glance, practical results are obtainable in spiral curve design for a transition to any circular curve usable for the intended vehicle speed rating of the highway. It is a simple graphical means of combining Eqs. 89, 90, and 16*b* for ready use.

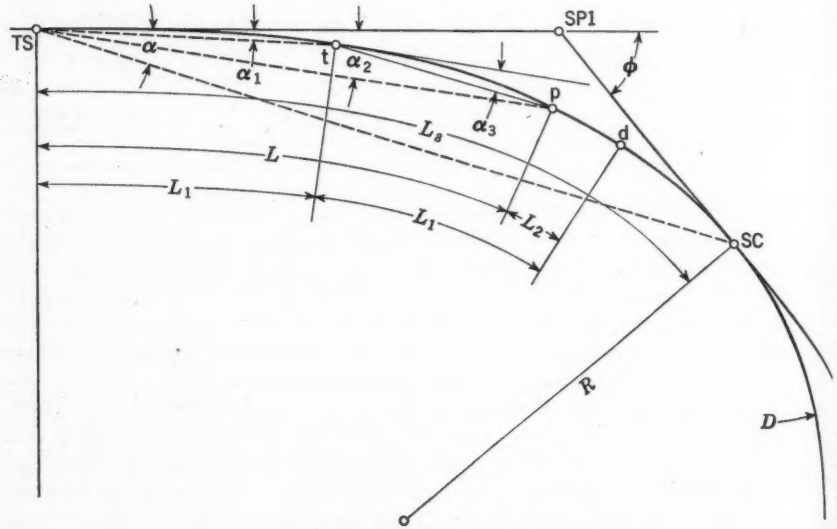

FIG. 16.—DEFLECTION ANGLES ON A SPIRAL CURVE

Messrs. Eichler and Eves expressed an interest in the possibility of improving on the present osculating circle method of laying out the spiral whenever the transit must be set up at any point, *t*, on the curve. This is a simple problem to solve.

Derivations will not be given here; but the deflection angle α_2 (in minutes) from the beginning of the spiral (TS, Fig. 16) to any point p is:

$$\alpha_2 = \frac{D\,L^2}{10\,L_s} - c \qquad (93)$$

in which D is the degree of circular curve connected to the spiral curve at a distance L_s (feet) from the TS; and L is the distance in feet along the spiral

curve from the TS to point p. A table of corrections c, in minutes, corresponding to the given spiral angles ϕ, in degrees, at 0.1 increments of the total spiral length, beginning at 0.3 and ending at 1.0 of the spiral length, is available (59).

The deflection angle from the tangent to the spiral curve at any transit point, t, to any point, p, is equal to the deflection calculated from the TS to that point, plus a straight-line correction. This correction is:

$$C' = \frac{D\,L_1\,L_2}{10\,L_s} \qquad (94)$$

The constant $\frac{D}{10\,L_s}$, as used in the formula for the deflection from the TS to point p, is also used in this expression. The length L_1 is the distance in feet from the TS to the transit, and L_2 is the distance, in feet, from a point d to the deflection point p. Point d is at a distance L_1 in feet, from the transit in direction of the sharpest curvature. When distance L_2 to point p is measured from point d toward the TS it is negative, and when measured from point d in a direction away from TS the distance L_2 is positive. This fact must be remembered when calculating deflections for any transit position not on the TS.

The simple formula—

$$\alpha_3 = \frac{D\,L^2}{10\,L_s} + \frac{D\,L_1\,L_2}{10\,L_s} - c \qquad (95)$$

—can be used to solve the minutes of deflection from any transit setup to any deflection point. In field work, it is customary first to calculate the α_2-deflections from the TS to all spiral curve points, and to record them in the field notebook. When it is necessary to move the transit forward the simple corrections are easily made.

If the length of spiral L_s is divisible into the degree of circular curve D an even number of times the constant $\frac{D}{10\,L_s}$ and the constant $\frac{D\,L_1}{10\,L_s}$ are simple decimal fractions with definite digit endings, which is of considerable advantage to the transitman. This point can be easily kept in mind when the spiral curve is designed.

The paper has served to assemble, in one place, many of the salient points regarding the clothoid (Euler's spiral), which is a curve that follows an exact law. Approximations should be avoided as much as practicable because the inexperienced engineer will eventually be embarrassed when he uses approximate formulas where more exact results are required. Once tables have been prepared on an exact basis, the labor of calculating geometric parts (functions) of the spiral curve by the use of long-series equations, or by using approximations of such equations, has been eliminated. Then the office and field work of designing and staking spiral curves as transitions in highway alinement becomes simple and easy.

Bibliography.—

(2) **"Transition Curves for Highways,"** by Joseph Barnett, Bureau of Public Roads, U.S.D.A., Government Printing Office, Washington, D.C., 1938. (*a*) Table IV.

(19) **"Skidding Characteristics of Automobile Tires on Roadway Surfaces, and Their Relation to Highway Safety,"** by R. A. Moyer, *Bulletin No. 120*, Iowa Eng. Experiment Station, Ames, Iowa, 1934.

(27) **"A Practical Method for the Improvement of Existing Railway-Curves,"** by W. H. Shortt, *Proceedings*, Inst. C. E., Vol. 176, 1909, p. 97.

(29) **"The Railway Transition Spiral,"** by A. N. Talbot, 6th Ed., Eng. News Publishing Co., 1927. (*a*) p. 93.

(59) **"Circular Curve and Unit Length Spiral Tables,"** by William T. Pryor, copyright 1942.

AMERICAN SOCIETY OF CIVIL ENGINEERS

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DISCUSSIONS

DETERMINATION OF PRESSURES WITHIN A SIPHON SPILLWAY

Discussion

BY I. M. NELIDOV

I. M. NELIDOV,[14] M. AM. SOC. C. E.[14a]—The discussion by Mr. Shukry is constructive and positive in its conclusions. On the basis of experiments, it is demonstrated that a similarity exists between the actual and the theoretical velocity, with variations on the safe side—that is, theoretical velocities are always smaller than actual velocities by various percentages ranging from 5% to 14% (see Table 3 and Figs. 4 and 5). Mr. Shukry makes a valuable comparison between the theoretical and actual discharges for seven existing siphons, which indicates that theoretical discharges are always smaller by various percentages, ranging from zero to 16%. On one occasion (for the Ferro siphon) the percentage variation was as high as 46%, which leads Mr. Shukry to conclude that for large ratios of $\frac{r_c}{r_i}$ (greater than 3.0) the discrepancy increases.

With decreasing values of r_i a state may be reached such that the velocity V_i at the summit will be the limiting value based on the available atmospheric vacuum, after which the discharge will not be appreciably increased. The writer wishes to refer to his discussion of the article by Elmer Rock,[15] Assoc. M. Am. Soc. C. E., in which he indicated the possibility of a "limiting" discharge and a corresponding "limiting" head with the inference that further increase of the head will not produce increase in discharge. Similar results may be obtained with a constant head by decreasing the radius r_i. Referring to Example 1 and Table 1 of the paper, it may be found that by making $\frac{p_i}{\gamma} = 0$ in Eq. 3 and with $h_a = 24$ ft, $h = -10$ ft, and $h_{vm} = 1.55$ ft:

$$h_{vi} = h_a + h + n\,h_{vm} = 14 + n\,h_{vm} \quad \ldots\ldots\ldots\ldots\ldots\ldots (33)$$

in which (see Eq. 11) $n = \dfrac{\beta^2\,(m^2 - 1)}{2 \log_e m}$.

NOTE.—This paper by I. M. Nelidov was published in March, 1945, *Proceedings*. Discussion on this paper has appeared as follows: December, 1945, by A. Shukry.

[14] Civ. and Structural Engr., El Cerrito, Calif.

[14a] Received by the Secretary March 18, 1946.

[15] Discussion by I. M. Nelidov of "Design of High-Head Siphon Spillway," by Elmer Rock, *Transactions*, Am. Soc. C. E., Vol. 105 (1940), p. 1069.

On the other hand, by Eq. 18, $h_{vi} = \beta^2 m^2 h_{vm}$. Equating Eqs. 18 and 33 and eliminating h_{vi}: $14 + n h_{vm} = \beta^2 m^2 h_{vm}$; or $-n + \beta^2 m^2 = \frac{14}{h_{vm}} = \frac{14}{1.55} = 9$.

By Eqs. 11 and 15 and with $m = \frac{r_c}{r_i}$:

$$\beta^2 \left[-\frac{m^2 - 1}{2 \log_e m} + m^2 \right] = 9 \quad \ldots\ldots\ldots\ldots\ldots\ldots (34a)$$

and

$$\left(\frac{d}{r_c}\right)^2 \frac{1}{\log_e^2 \frac{r_c}{r_i}} \left[-\frac{\left(\frac{r_c}{r_i}\right)^2 - 1}{2 \log_e \left(\frac{r_c}{r_i}\right)} + \left(\frac{r_c}{r_i}\right)^2 \right] = 9 \quad \ldots\ldots\ldots (34b)$$

With $d = 4$ ft and $r_c = 20$ ft, Eq. 34*b* becomes

$$\frac{0.04}{\log_e^2 \frac{20}{r_i}} \left[-\frac{\left(\frac{20}{r_i}\right)^2 - 1}{2 \log_e \left(\frac{20}{r_i}\right)} + \left(\frac{20}{r_i}\right)^2 \right] = 9 \quad \ldots\ldots\ldots (34c)$$

By trial it is found that a value of $r_i = 0.3$ ft will be the limiting radius of the invert. It is true that the value of the ratio $\frac{r_c}{r_i} = \frac{20}{0.3} = 67$ is so great that the theoretical results may be questioned, but this example was used only as a numerical illustration of the process of obtaining r_i. At the same time the small value of r_i demonstrates that in this example the flow will agree with the assumed theory of irrotational vortex.

Another point of considerable interest was treated by Mr. Shukry in his discussion of the mean pressure, velocity, and position heads corresponding to the nonuniform distribution of velocities. As shown by Fig. 1, the writer has related his energy and hydraulic gradient lines to their mean values and not to those corresponding to the mean velocity.

The use of losses in the siphons computed with the coefficients selected from nonrelated tables should give quite divergent results. However, the main fault is the scarcity of information pertaining specifically to the siphon flow rather than defective experiments.

The writer wishes to thank Mr. Shukry for his exhaustive study of the problem.

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DISCUSSIONS

THE SAFETY OF STRUCTURES

Discussion

BY NOMER GRAY

NOMER GRAY,[39] Assoc. M. AM. SOC. C. E.[39a]—It is inevitable that the attention of structural engineers will be directed with increasing frequency to a re-examination of the entire subject of factors or margins of safety. When consideration is given to the profound part played by this factor in the design of structures, the wonder is that it has not been continuously the subject of discussion. Certainly the device of using "working stresses," although perfectly justifiable on the ground of expediency, has diverted attention from this most important element of design. However, the true reason is the inherent complexity of the problem—a complexity which becomes more evident from a study of this paper.

Studies leading to refinements in the methods of stress calculation have persisted at a high level of activity for years, and devising new variations on old and tried methods of stress calculation can be classed as an occupational ailment. Although it leaves much to be desired, current knowledge of materials has been enhanced greatly since the 1920's, thus materially reducing this element of "uncertainty" in design. These two statements do not necessarily sponsor higher working stresses, but rather question the validity of a uniform factor of safety for a great variety of members in a structure regardless of function.

An extreme example may be found in the selection of the required sectional area for the main cables of suspension bridges. The three "uncertainties" requiring the use of a safety factor may be simply stated as: (1) Accuracy of method of stress calculation; (2) uniformity of material; and (3) present and future validity of the design loading.

In regard to uncertainty (1), few members in the entire field of structures inspire as little doubt as to the correctness of the computed stress. With respect to uncertainty (2), the material used in the cables of large bridges is

NOTE.—This paper by Alfred M. Freudenthal was published in October, 1945, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: January, 1946, by F. H. Frankland, and Elliott B. Roberts; and February, 1946, by A. G. Pugsley, and Lynn Perry.

[39] Civ. Engr., Dept. of Public Works, City of New York, New York, N. Y.

[39a] Received by the Secretary February 28, 1946.

remarkably uniform. Taking the wire used in the George Washington Bridge cables as an example, the variation in yield point is recorded[40] as from 170 kips per sq in. to 189 kips per sq in.—a spread of 10%; and the variation in ultimate strength is from 225 kips per sq in. to 240 kips per sq in.—only 6.4%. These are extremes. Actually, the separate values cluster closely around the mean (the standard deviation is small).

It is in respect to uncertainty (3), however, that this member (the cable) is almost unique. For long-span highway bridges, the ratio of dead to live load is frequently 5 to 1. There is little uncertainty about the design dead load and an excess of 10% may be regarded as extreme if the weight is calculated with moderate care. Any major increase in the dead load arising from alterations made during the life of the structure is certain to occur only after an analysis, and the justification of the increase can be decided at that time. There is very little uncertainty regarding the dead load.

An increase of 100% in the live load will only produce an approximate 17% increase in the cable stress; and such an increase could not take place without adequate warning, for the stiffening system would be seriously overstressed (and perhaps fail) before this large increase in live load could occur.

In spite of the uniquely certain character of material and stress in the cables, however, the working stress is commonly set at 80 kips per sq in. This gives a factor of safety of approximately 2.1 based upon the least value of yield point and 2.8 if based upon ultimate strength; and, as the cable wire does not have a pronounced yield point, the latter value is more significant. In the light of the foregoing, some decrease in the factor of safety appears entirely justifiable.

In view of the firmly established and completely accepted meaning associated with the word "strain," its use in a different sense by the author is unfortunate. The same may be stated of the definitions of factor of safety as contained in Eqs. 6*a* and 6*b*. A "factor" of safety is commonly expressed as an abstract number indicating a ratio, whereas these equations apparently express "margins of safety."

Throughout the paper the use of the technique and nomenclature of the mathematics of statistics is evident. The statistical approach alone cannot give the answer, for the determination of a factor of safety involves questions of a philosophical nature. Statistics can present the facts upon which a judgment may be made, however. The author is to be congratulated for having so fully developed the statistical approach to this inherently complex and difficult problem.

Correction for Transactions: In October, 1945, *Proceedings*, page 1167, Eq. 13, the lower limit of the integral should be 0 rather than $-\infty$, as the lower limit of tensile strength cannot be less than zero.

[40] "George Washington Bridge: Materials and Fabrication of Steel Structure," by Herbert J. Baker, *Transactions*, Am. Soc. C. E., Vol. 97 (1933), p. 366, Table 9.

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DISCUSSIONS

BALANCED DESIGN IN URBAN TRIANGULATION

Discussion

BY WALTER S. DIX

WALTER S. DIX,[6] Assoc. M. Am. Soc. C. E.[6a]—The paper is a comprehensive analysis of a basic problem in urban area precision survey engineering, important to many surveyors and engineers, yet understood or considered by an alarming few. The author's research and effort on this paper are valuable contributions to the knowledge and advancement of professional surveying and engineering.

It is imperative that, for the degree of accuracy warranted by modern municipal precision surveys, not one single step in the direction of equilibrium for final values of an urban network should be neglected—that, whereas attention has been given in the past to full consideration of the laws of gravity, refraction, and trigonometry in city triangulation, not enough thought has been applied to equally important laws of balanced design, involving engineering philosophy with mathematical, geometrical, and algebraic analysis, to produce truly homogeneous accuracy.

In substance, the theory of homogeneity in this instance is that the reality of engineering has been added to the science of mathematics to produce the surprising data in the author's tabulation of final results.

The author is completely justified in comparing localization of error in an urban triangulation network, to the inherent weakness of a similarly localized error in structural design; and, as the author has indicated, experience shows that arbitrary acceptance of over-all closure adjustment in triangulation schemes heterogeneously composed of unsymmetric figure patterns of varying strength, as true index of homogeneous accuracy of the scheme, can quite conceivably conceal some local accrual of weakness.

The possibly of such concealment of local error in over-all closure values has previously been mentioned by G. D. Whitmore, M. Am. Soc. C. E.,

NOTE.—This paper by Charles D. Hopkins was published in November, 1945, *Proceedings*.

[6] Cartographic Engr., Maps and Surveys Div., TVA, Washington, D. C.

[6a] Received by the Secretary March 6, 1946.

C. C. Miner, Assoc. M. Am. Soc. C. E., and W. O. Byrd who state:[7]

> "The writers have long felt that too much emphasis has been placed on the 'just happened' closure of the entire loop, and not enough on designing measuring instruments and measuring procedures that will insure uniform precision for each and every tangent of the entire traverse. Experience shows that traverses having circuit closures of high order, such as 1:10,000 or 1:20,000, may conceal errors, in individual courses, of the order of 1:1,000."

This statement follows closely the author's argument in favor of closer unit consideration in design, and solution of urban triangulation systems, to insure greater integral accuracies within the accepted over-all accuracy index.

Any precautionary measure taken toward eliminating the possible concealment of error is a step in the direction of optimum engineering design. The deliberate design of urban triangulation by compounding component unit figures, and the analytical selection of balanced equations for homogeneous solution with multiple determination alternatives, as propounded by the author (as compared with the over-all solution of an accidental heterogeneous triangulation pattern resulting only by intervisibility from conveniently available tall buildings, hill tops, tanks, etc.), is certainly a step in that direction. Furthermore, station towers should even be erected where required when found advantageous for optimum design strength in following out the overlapping unit-figure network in urban triangulation.

Portable towers are used by the United States Coast and Geodetic Survey to aid visibility and to help control figure strength in basic triangulation arcs of the United States; but, to the writer's knowledge, erected towers have seldom if ever been specially placed for urban triangulation, except scaffolding or platforms built up to augment convenient roof structures.

With reference to the author's closing statement, any latently discovered weakness in an urban triangulation system requiring rationalizing by arbitrarily adjusting the values of subsequent determinations is more than disheartening disappointment; it is an engineering tragedy. Urban networks, by characteristic unity, are capable of complete predesign. Other than false economy or meager budget (itself a false economy), there seems little argument contrary to urban triangulation network predesign.

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[7] *Transactions*, Am. Soc. C. E., Vol. 108 (1943), p. 598.

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DISCUSSIONS

CAVITATION IN HYDRAULIC STRUCTURES
A SYMPOSIUM

Discussion

BY JOHN S. MCNOWN

JOHN S. MCNOWN,[10] ASSOC. M. AM. SOC. C. E.[10a]—Investigations of cavitation, as indicated by the Symposium authors, have been restricted for the most part to laboratory and field observations of the factors causing cavitation and the countermeasures required for prevention. Although it is frequently possible to predict and eliminate cavitation on the basis of experience and model studies, very little is known concerning the thermodynamics of the local vaporization and the subsequent condensation of the water, the dynamics of the bubble collapse, or the manner in which boundary surfaces are pitted. Fundamental research, in time, should reveal much of this information and lead to a basic understanding of this phenomenon, but for the present it is essential that the available tools be utilized to their utmost efficiency. The writer wishes to discuss two of these tools—analytical methods, which he feels were unduly slighted in the Symposium, and the variable-pressure water tunnel.

For certain types of problems the assumption of negligible viscosity and the application of the corresponding equations of flow yield very worthwhile results, particularly since cavitation occurs in or immediately downstream from a zone of accelerated flow. Therefore, some of the statements made by Professor Vennard concerning the analytical approach are definitely misleading. The criterion for the applicability of the Bernoulli equation is not the absence of curvilinear flow, but rather the extent to which the flow can be described by the equations of irrotational motion. For example, it is obviously impossible to apply the Bernoulli equation across rectilinear laminar flow in a pipe line, whereas the familiar vortex (Fig. 1) is called the irrotational vortex because of the excellent agreement between theory and measurement for flow which is wholly curvilinear. To be sure, uncertainties in the magnitude of the pressure,

NOTE.—This Symposium was published in September, 1945, *Proceedings*. Discussion on this Symposium has appeared in *Proceedings*, as follows: December, 1945, by C. A. Mockmore; February, 1946,, by Joseph N. Bradley; and March, 1946, by J. M. Robertson, and Fred W. Blaisdell.

[10] Research Eng., Iowa Inst. of Hydr. Research and Asst. Prof., Dept. of Mechanics and Hydraulics State Univ. of Iowa, Iowa City, Iowa.

[10a] Received by the Secretary March 14, 1946.

which are of importance in cavitation studies, arise in the vortex core; however, these uncertainties are caused by departures from the irrotational pattern, and not by the presence of curvilinear motion.

Irrotational motion is also approached very closely in regions of rapidly accelerated flow of the type indicated in Figs. 1(*a*) and 1(*b*). In fact, the two-dimensional counterpart of Fig. 1(*a*)—the horizontal slot—has been solved by the methods of classical hydrodynamics with remarkably close agreement between theory and experiment. In connection with tests in the variable-pressure water tunnel at the Iowa Institute of Hydraulic Research in Iowa City, the writer was able to predict, entirely by analytical means, the conditions for incipient cavitation on a mathematically defined boundary form. Hence, to describe all curved-flow problems (see heading, "Occurrence of Cavitation") as "most obscure and unpredictable" not only conveys a very false impression, but also completely ignores one useful avenue of approach.

Admittedly, only part of the problems encountered in engineering practice can be solved by analytical methods alone, and as a result many hydraulic structures can best be designed from model studies. Thus, consideration should be given to the methods of analysis and the experimental procedure for model studies of cavitation effects. For such a study, any dependent variable can be described by a function of viscous, gravitational, and cavitation effects, together with the appropriate description of boundary geometry. In general terms,

$$\mathbf{D} = f(\mathbf{R}, \mathbf{F}, \mathbf{K}, b/a, c/a, \cdots) \quad (5)$$

in which **D** is any dependent variable in dimensionless form; **R** and **F** are the familiar Reynolds and Froude numbers; **K** is a cavitation parameter, the definition of which depends in part on the type of problem under consideration; and b/a, c/a, etc., are ratios of linear dimensions.

Three phases of cavitation research are of importance in engineering practice, and the general equation may be adapted to each of these. The first phase, and perhaps the most fundamental, is that of flow without a free surface such as flow past a submerged body or flow through a siphon spillway. Since viscous effects may be significant, the dependent variable can be expressed in terms of the Reynolds number, a cavitation parameter, and the boundary geometry:

$$\mathbf{D} = f_1(\mathbf{R}, \mathbf{K}, b/a, c/a, \cdots) \quad (6)$$

The dimensionless parameter **D** may describe in relative terms a pressure drop, loss of head, length of cavitation pocket, amount of pitting which takes place, or some other pertinent quantity. The cavitation parameter is indicative of the tendency toward cavitation, and hence is determined by the normal center-line pressure p_0, the vapor pressure p_v, the undisturbed velocity v_0 and the density ρ. That is,

$$\mathbf{K} = \frac{p_0 - p_v}{\frac{1}{2}\rho v_0^2} \quad (7)$$

and a reduction in **K**, resulting from either an increase in velocity or a reduction in pressure, increases the tendency toward cavitation. In most instances

where cavitation takes place, the Reynolds numbers are large and the influence of viscosity is correspondingly slight. It must be kept in mind, however, that viscosity may play an important rôle should the model scale or velocity be too small.

The second type of problem is that of flow with a free surface for which the pattern of motion is completely described by the boundary geometry, as in flow over a spillway section. Since the Froude number cannot be varied at will once the head H and the boundary geometry are determined, $\mathbf{F}$ is not an independent variable for this type of flow; and, therefore,

$$\mathbf{D} = f_2(\mathbf{K}', a/H, b/H, \cdots) \quad (8)$$

The simulation of prototype cavitation conditions in a model spillway requires the reduction of the atmospheric pressure p_a in the model, and the cavitation parameter for this type of flow may be written

$$\mathbf{K}' = \frac{p_a - p_v}{\rho g H} = \frac{p_a - p_v}{\gamma H} \quad (9)$$

A third type of problem is that of flow with a free surface in which the velocity of flow can be varied independently of the depth, as in a study of cavitation effects on baffle piers. The added independent variable necessitates the inclusion of the Froude number, and the depth of approaching flow y replaces H as the linear dimension of the flow. Hence,

$$\mathbf{D} = f_3(\mathbf{F}, \mathbf{K}, a/y, b/y, \cdots) \quad (10)$$

in which a, b, etc., are linear dimensions of the pier. The cavitation parameter may take either of the two alternative forms:

$$\mathbf{K} = \frac{p_a - p_v}{\frac{1}{2}\rho V^2} \quad (11a)$$

or

$$\mathbf{K}' = \frac{p_a - p_v}{\rho g y} \quad (11b)$$

which are related by the Froude number.

For any pattern of flow, such a particular value of $\mathbf{K}$ exists that for all higher values the flow is unaffected by cavitation, just as the pattern of flow past a body tends to become independent of viscous effects if $\mathbf{R}$ exceeds a characteristic magnitude. If the value for incipient cavitation is denoted by $\mathbf{K}_i$, cavitation effects commence for $\mathbf{K} = \mathbf{K}_i$ and become more and more pronounced as $\mathbf{K}$ is reduced below the critical. Furthermore, for a given value of the cavitation parameter, essentially similar cavitation phenomena will be found for various combinations of velocity and pressure on both model and prototype. It follows that Eqs. 2, 3, and 4 in Mr. Harrold's paper—which may be written in the form,

$$(p_a - p_v)_m = L_r(p_a - p_v)_p \quad (12)$$

—are obtained if identical values of $\mathbf{K}$ are specified for model and prototype. That is, if $\mathbf{K}_m = \mathbf{K}_p$:

$$\frac{(p_a - p_v)_m}{\gamma H_m} = \frac{(p_a - p_v)_p}{\gamma H_p} \quad (13)$$

and Eq. 12 is thereby obtained, provided the same liquid is used in both model and prototype. However, Eq. 3 does not completely determine **K** unless the Froude number depends on the boundary geometry. It is evidently not sufficiently general for use, say, on the baffle-pier study just discussed.

The problem of flow over a spillway section, discussed by Mr. Harrold, may be used further to illustrate an approach to the experimental determination of cavitation effects and to emphasize the important rôle of the cavitation parameter. If the discharge per unit width q is taken as the dependent variable, Eq. 8 takes the form,

$$\frac{q}{\sqrt{g}\,H^{3/2}} = C_d = f(\mathbf{K}', a/H, b/H, \cdots) \quad \ldots\ldots\ldots\ldots\ldots\ldots (14)$$

For all values of **K** greater than $\mathbf{K}_i$, the variation of C_d with H, as determined by experiment, would be entirely independent of cavitation effects in this region, and would therefore be represented by the uppermost curve in Fig. 55. However, the onset of cavitation near the spillway crest would substantially reduce C_d, so that further tests at values of **K** less than $\mathbf{K}_i$ would yield a series of curves such as that shown schematically in Fig. 55. The lower the value of **K** or the greater the degree of cavitation, the greater will be the departure from cavitation-free conditions. In a similar manner, the effect of cavitation upon other dependent variables could be investigated.

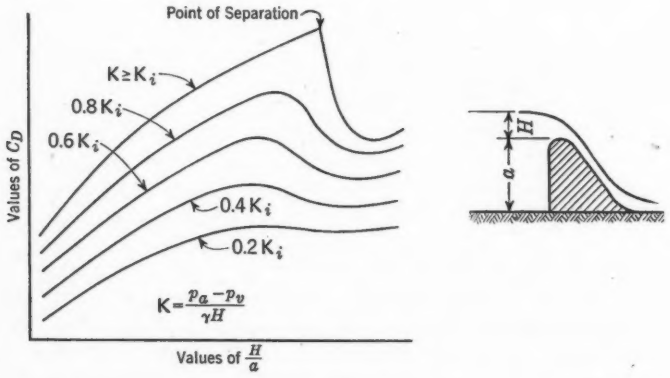

FIG. 55.—SCHEMATIC REPRESENTATION OF CAVITATION EFFECTS ON THE DISCHARGE COEFFICIENT OF A SPILLWAY

Even though it is impossible to make a model study which satisfies both the Froude and Reynolds criteria if the same fluid is used in both model and prototype, no comparable obstacle prevents the simultaneous study of cavitation phenomena and either viscous or gravitational effects if it is possible to vary the pressure on the system. As a result, much of the laboratory work on cavitation has been performed in variable-pressure water tunnels, of which more than a dozen have been built in research laboratories throughout the world. Not unlike closed-circuit wind tunnels in design, water tunnels have been used to test the cavitation characteristics of ship screws, turbine blades,

and a variety of two-dimensional and three-dimensional body forms. In many respects the water tunnel is similar to the apparatus developed at Carnegie Institute of Technology in Pittsburgh, Pa., and described by Messrs. Vennard and Harrold.

Although water tunnels can be used for the variety of studies already mentioned, to date (1946) the experiments in the variable-pressure water tunnel at the Iowa Institute of Hydraulic Research have been devoted exclusively to the measurement of the effect of cavitation on the pressure distribution around underwater bodies. Nevertheless, general conclusions from these experiments concerning the effect on cavitation phenomena of viscosity and air content are applicable to all investigations of cavitation. Measurements of the pressure distribution around a given boundary form, which were taken at different Reynolds numbers and for values of **K** both above and below $\mathbf{K}_i$, indicated that viscous effects influenced the results for relatively small values of **R** and for low intensities of cavitation. This effect was found to be exaggerated for boundary curvatures of short radius, extending in one instance to Reynolds numbers of approximately 10^6.

There is usually a large amount of air dissolved in water, and the reduction in pressure that accompanies any tendency toward cavitation will release part of this dissolved air with much the same action as that involved in the formation of water vapor in zones of cavitation. Furthermore, because the pressures are low, the air released from solution may occupy a disproportionately large volume. Since it was considered likely, therefore, that the results of a study of cavitation might be affected by the quantity of air in solution, a series of tests was performed to determine the extent of this effect. The pressure was reduced steadily with a vacuum pump until an operational limit was reached below which the pump was unable to reduce the pressure in a reasonable period of time. With reference to this readily available minimum pressure, which was usually about 1.5 lb per sq in. above the vapor pressure, it was found that tests performed at pressures only slightly (0.2 lb per sq in. or less) above this operational minimum were affected by air coming from solution in such a manner as to produce results comparable with those for smaller **K**-values. For tests performed at pressures well above (0.4 lb per sq in. or more) this operational minimum, on the other hand, no effects of dissolved air were found.

For investigations directed toward the elimination of cavitation, model studies need not be conducted in variable-pressure chambers. The distribution of pressure along a boundary as determined in atmospheric-pressure models can be used to predict the tendency toward cavitation as shown by the studies at the Vicksburg (Miss.) laboratory. However, caution should be exercised in the interpretation of model results obtained in this manner, not only because atmospheric-pressure models may indicate impossibly low prototype pressures, but also because cavitation effects may be found even though the pressure on the boundary is everywhere well above the vapor pressure. Since cavitation first occurs in the fine eddies at the surface of separation, the pressure at a boundary surface a short distance away may be affected by the resulting change in the flow pattern. In one series of experiments at the Iowa

Institute of Hydraulic Research on flow past an abrupt boundary transition, although the minimum pressure and the vapor pressure were equal when $K = 0.7$, cavitation effects did not vanish until the value of $K = 1.8$ was reached. In other words, even though the pressure head at the boundary was more than one velocity head above the vapor-pressure head, the pressure distribution along the boundary was affected by local cavitation a short distance away.

In conclusion the writer wishes to welcome the Symposium on Cavitation in Hydraulic Structures as a worthwhile contribution to the summarization of the available information in this relatively new field, and to concur heartily with the Symposium authors in their statements concerning the need for publication of other available information and for stimulation of further research in this field.